DET FORSTLIGE FORSØGSVÆSEN I DANMARK

THE DANISH FOREST EXPERIMENT STATION
STATION DE RECHERCHES FORESTIÈRES DE DANEMARK
DAS FORSTLICHE VERSUCHSWESEN IN DÄNEMARK

BERETNINGER UDGIVNE VED DEN FORSTLIGE FORSØGSKOMMISSION

REPORTS — RAPPORTS — BERICHTE

BIND XXXIV

HÆFTE 4

INDHOLD

KØBENHAVN
TRYKT I KANDRUP & WUNSCH'S BOGTRYKKERI
1976

THE INTERRELATION BETWEEN BASIC DENSITY AND RING WIDTH OF NORWAY SPRUCE

SAMMENHÆNGEN MELLEM RUMTÆTHED OG ÅRRINGSBREDDE HOS GRAN

BY

P. O. OLESEN

1. INTRODUCTION

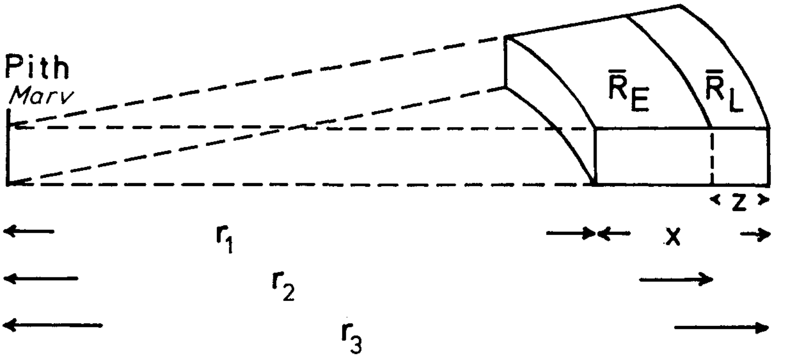
It is well established that the basic density of Norway spruce is strongly, negatively correlated with the ring width (*Klem* 1934, *Nylinder* 1953, *Ericson* 1960, *Bernhart* 1964, *Hakkila* 1966 and *Olesen* 1973). This negative correlation is mainly a result of the decrease in latewood percentage with increasing ring width (*Bernhart l.c.*). Therefore, when investigating the influence of various factors on the basic density, ring width must be taken into consideration in case the factor in question affects the ring width. This can be done, for example, by a multiple regression analysis or by comparing basic density/ring width curves for different factors. In the following a basic density/ring width curve is termed: *density level,* as the curve may assume different levels with the variation of a factor.

In previous investigations into the effect of various factors on basic density I have found it suitable to make an initial analysis of the effect of these factors one by one comparing density levels. For example, the effect of the height of the tree is assessed by comparing density levels from different heights of the tree. In such an analysis it is necessary to work with a model expressing the interrelation between basic density and ring width. The aim of this study is to derive such a model — a model which is not only an empirical description of the interrelation between the two variables, but one which also expresses some of the causal relationships.

2. DERIVING A MODEL

Latewood percentage is the factor with which the basic density of Norway spruce is most strongly correlated (*Bernhart* 1964). However, as the latewood percentage is also strongly correlated with the ring width, a strong correlation between basic density and ring width is also found. Furthermore, while latewood percentage is difficult to estimate accurately, ring width can easily be precisely determined. Thus, although ring width as such cannot affect the basic density, ring width is selected because it is a suitable independent variable which has the property of reflecting factors which have a direct effect on the basic density, such as latewood percentage, rainfall in the growing season, soil type etc.

An annual ring can be divided into earlywood and latewood, cf. Figure 1.



F i g. 1. Section of annual ring divided into earlywood and latewood. $\bar{R}_E$ = mean basic density of earlywood, and $\bar{R}_L$ = mean basic density of latewood.

F i g. 1. Sektion af årring opdelt i vår- og høstved. $\bar{R}_E$ = gennemsnitlig rumtæthed for vårved, og $\bar{R}_L$ = gennemsnitlig rumtæthed for høstved.

From the figure it can be seen that the basic density R can be expressed as:

$$R = \frac{\bar{R}_E(\pi r_2^2 - \pi r_1^2) + \bar{R}_L(\pi r_3^2 - \pi r_2^2)}{\pi r_3^2 - \pi r_1^2}$$

$$= \frac{\bar{R}_E(r_2 - r_1)(r_2 + r_1) + \bar{R}_L(r_3 - r_2)(r_3 + r_2)}{(r_3 - r_1)(r_3 + r_1)}$$

As

$$r_2 - r_1 = x - z,$$
$$r_3 - r_2 = z, \text{ and}$$
$$r_3 - r_1 = x$$

where x = ring width, and z = latewood width, we get

$$R = \frac{\bar{R}_E(x - z)(r_1 + r_2) + \bar{R}_L z(r_2 + r_3)}{x(r_1 + r_3)} \qquad (1)$$

As the relative differences between the three sums $r_1 + r_2$, $r_2 + r_3$, and $r_1 + r_3$ decrease rapidly with increasing distance from the pith, and as these differences are less than one per cent for a complete mature tree, the three sums can be ignored, so that equation (1) can be written as

$$R = \frac{\bar{R}_E(x - z) + \bar{R}_L z}{x} \qquad (2)$$

Furthermore, if the juvenile wood (i.e. the inner annual rings centred on the pith) is omitted, the error made in equation (2) is negligible.

In deriving equation (2), only one annual ring or a fraction of an annual ring was considered. However, it is possible to generalize equation (2) in such a way as to be valid for both trees and populations of trees. In a pilot study of the dependence of the basic density of earlywood and latewood on ring number, it was found that the mean earlywood density, $\overline{R}_E$, decreases over the first rings, whereafter it remains relatively constant. The mean latewood density, $\overline{R}_L$, on the other hand, increases over the first 25 rings, after which it remains relatively constant, though with a slight tendency to decrease after ring no. 40—50, cf. Figure 2. These results are in agreement with earlier findings (*Panshin & de Zeeuw* 1970).

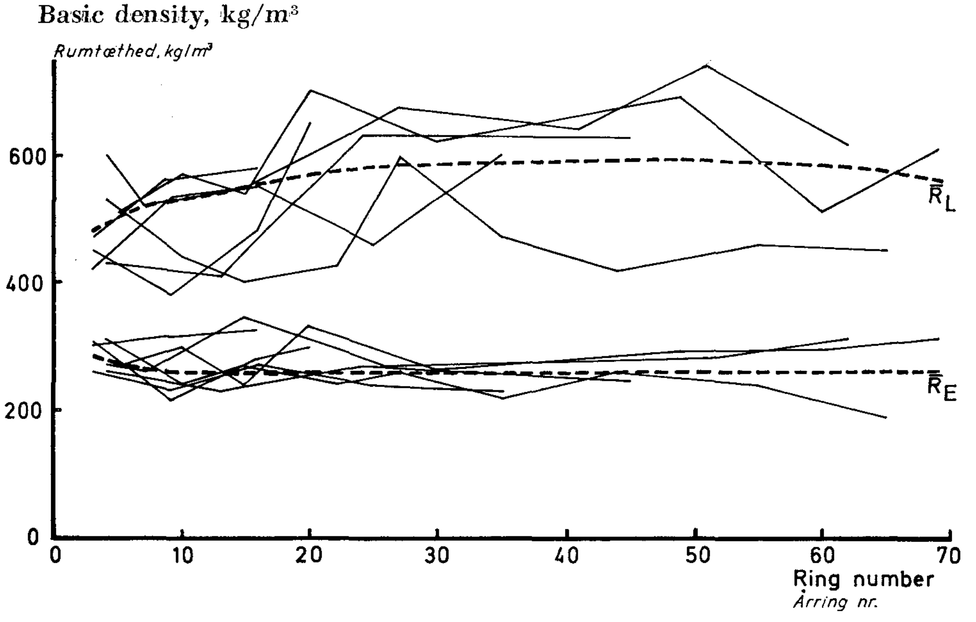


Fig. 2. The dependence of earlywood and latewood basic density on ring number. Each line represents the basic density of the earlywood or latewood in a tree. Dotted curves = smoothed means.

Fig. 2. Sammenhæng mellem årringsnummer og vår- og høstveddets rumtæthed. Hver linie repræsenterer vår- hhv. høstveddets rumtæthed i et træ. Stiplede kurver = udjævnede gennemsnit.

Figure 2 shows that, in adult wood, the average densities of earlywood and latewood seem to be independent of ring number. Thus, the two means represent the means of the earlywood and latewood densities at a certain height of the tree. If these means should vary with the height in the tree, the means in equation (2) then represent the means of the mean values at different heights, in other words the means of the earlywood and latewood densities for the whole tree. If more than one tree form part of the investigation, $\bar{R}_E$ and $\bar{R}_L$ then represent the means of all trees involved. The variation about the means, $\bar{R}_E$ and $\bar{R}_L$ is mainly caused by climatic differences from growing season to growing season. The mean values are therefore an expression of the genotypic values of the population for the environment in question.

Within the juvenile wood the basic density of both the earlywood and latewood changes with ring number so that each ring number has its own density level.

If the width of the latewood, z, can be expressed as a function of the ring width, equation (2) can be solved with respect to x, and the interrelation between basic density and ring width can be derived. Investigations by *Bertog* (1895), *Trendelenburg* (1936), *Johansson* (1939), *Nylinder* (1951), *Klem* (1957), and *Bernhart* (1964) show unequivocally that the percentage of latewood decreases with increasing ring width for Norway spruce. This is illustrated in Figure 3, which is based on the mean values presented by *Nylinder* (l.c.).

Such a relationship between latewood *percentage* and ring width results

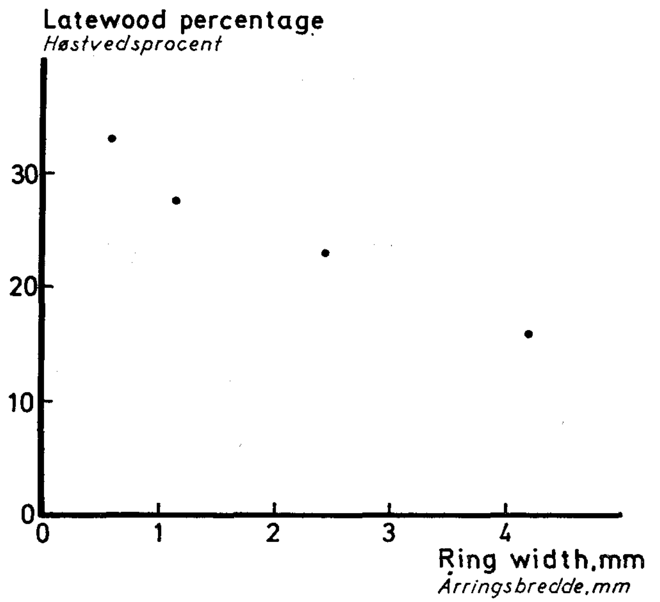


Fig. 3. Relationship between latewood percentage and ringwidth. From Nylinder (1951).
Fig. 3. Sammenhæng mellem høstvedsprocent og årringsbredde.

in an increase in latewood *width* with increasing ring width. This is illustrated in Figures 4 and 5, and it is seen that the curves drawn through the observations are curvilinear leveling off with increasing ring width. (These figures are based on data presented by *Nylinder* (l.c.) and *Klem* (l.c.), the original data being converted from latewood percentage to latewood width by the author). Furthermore, the curves → (0,0) for x → 0.

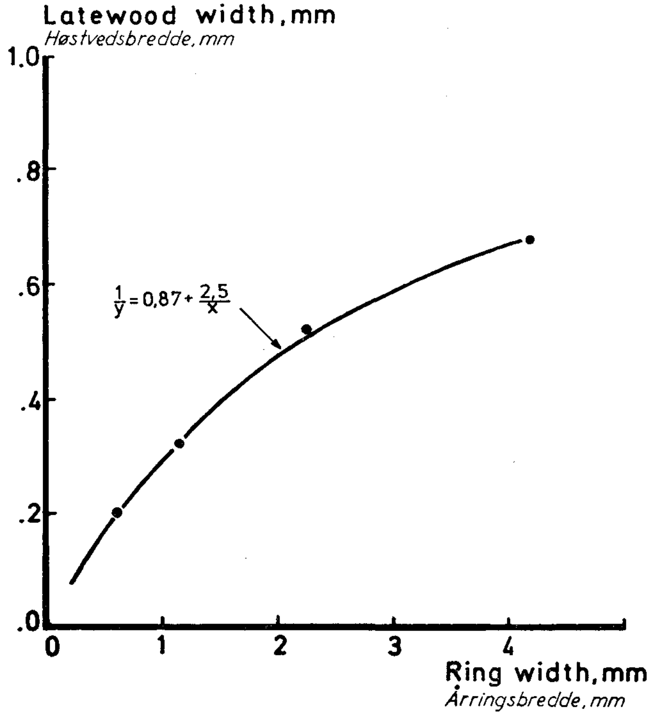


Fig. 4. Relationship between latewood width and ring width. The dots represent the means of 23, 33, 36, and 8 trees respectively (from left to right). From Nylinder (1951). The hyperbola 1/y = 0.87 + 2.5/x has been fitted by the author. The curve → (0.0) for x → 0.

Fig. 4. Sammenhængen mellem høstvedsbredde og årringsbredde. Punkterne repræsenterer gennemsnit af 23, 33, 36 og 8 træer fra venstre til højre.

The curves which fit the observations in Figures 4 and 5 resemble among others hyperbolas and parabolas. For example, Nylinder's data fit closely to the hyperbola $1/y = 0.87 + 2.5/x$, cf. Figure 4.

If the curves in Figures 4 and 5 are interpreted as hyperbolas, these types of hyperbolas are given by the formula

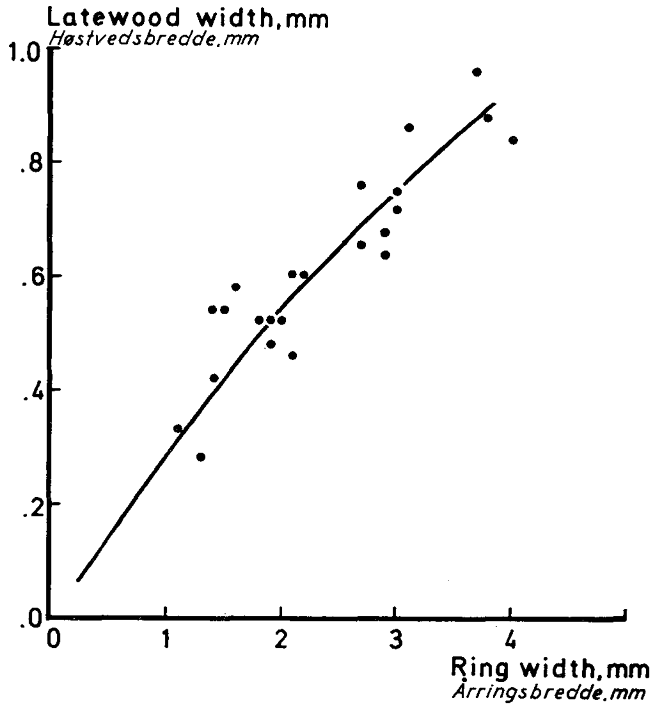


Fig. 5. Relationship between latewood width and ring width. Each dot represents the mean of a tree. From Klem (1957). The curve through the data has been fitted by the author. The curve → (0.0) for x → 0.

Fig. 5. Sammenhæng mellem høstvedsbredde og årringsbredde. Hvert punkt repræsenterer gennemsnit af et træ. Udjævningskurven tegnet af forfatteren.

$$1/z = u + v/x \text{ or } z = \frac{x}{ux + v}$$

where x = ring width, z = latewood width, and u and v are two positive constants.

If $z = x/(ux + v)$ is substituted in equation (2), the following equation is obtained:

$$R = \frac{\overline{R}_E \left(x - \frac{x}{ux+v}\right) + \overline{R}_L \left(\frac{x}{ux+v}\right)}{x}$$

$$= \overline{R}_E + \frac{\overline{R}_L - \overline{R}_E}{ux + v} \qquad (3)$$

Since $\overline{R}_E$, $\overline{R}_L$, u, and v are four constants, equation (3) can be written as

$$\boxed{R = a + \frac{b}{x + c}} \qquad (4)$$

where all three constants are positive and a is equal to the basic density of the earlywood.

If the curves in Figures 4 and 5 are taken as parabolas with vertex of $(-u, -v)$, we have

$$z + v = p\sqrt{x + u} \qquad (5)$$

As the parabola passes through the origin, i.e. $z = 0$ for $x = 0$, we have

$$p\sqrt{u} - v = 0 \quad \text{for } x = 0$$

or

$$v = p\sqrt{u}$$

which substituted in equation (5) gives

$$z = p\sqrt{x + u} - p\sqrt{u} = p(\sqrt{x + u} - \sqrt{u}) \qquad (6)$$

If equation (6) is substituted in equation (2),

$$R = \frac{\overline{R}_E(x - p(\sqrt{x + u} - \sqrt{u})) + \overline{R}_L p(\sqrt{x + u} - \sqrt{u})}{x}$$

$$= \overline{R}_E + \frac{p\,(\overline{R}_L - \overline{R}_E)\,(\sqrt{x + u} - \sqrt{u})}{x}$$

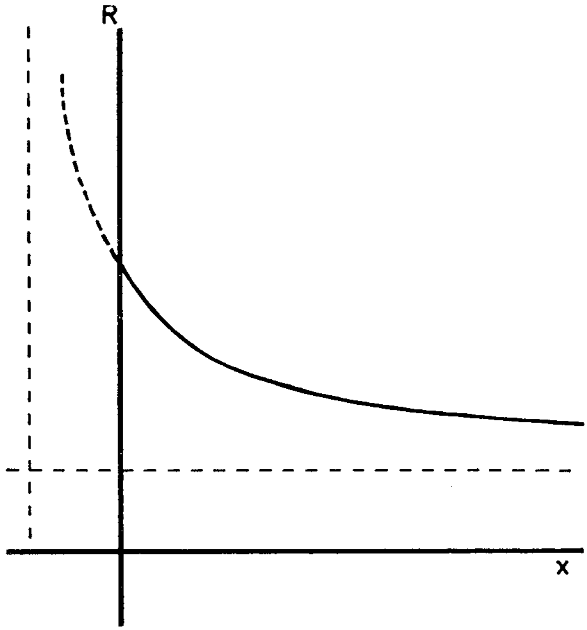
$$= \overline{R}_E + \frac{p(\overline{R}_L - \overline{R}_E)}{\sqrt{x + u} + \sqrt{u}} \qquad (7)$$

As $\overline{R}_E$, $\overline{R}_L$, p and u are four constants, equation (7) can be written as

$$\boxed{R = a + \frac{b}{\sqrt{x + c} + \sqrt{c}}} \qquad (8)$$

where all three constants are positive, and a is equal to the basic density of the earlywood.

The two derived models, equation (4) and (8), are very similar. They are both hyperbolas, with the horizontal asymptote $R = a$, cf. Figure 6.



F i g. 6. The graph of (4) and (8).
F i g. 6. Formlerne (4) og (8) afbildet grafisk.

3. MATERIAL AND METHODS

The material used originates from previous studies, and from investigations in progress concerning the basic density of Norway spruce, and includes 240 trees from different growth localities in Denmark.

In most cases the basic density is determined using 4.2 mm increment cores taken at breast height. All samples are saturated with water and then examined for defects over a light box. All annual rings with defects are cut out and discarded. As increment cores often contain several annual rings with defects, especially compression wood, a large proportion of the material frequently has to be discarded. If the material is rather small, a relatively large percentage of defects could be detrimental to the investigation, so that, in order to utilize the available material to best advantage, the following technique has been adopted whenever possible. A disc, at least 3 cm thick, is cut out from each tree and brought to the laboratory. Each disc is examined for defects, and a stick, 6—8 mm thick (axially), and 8—10 mm wide (tangentially) is cut out from pith to bark from a faultless section of the disc. In this way the amount of defects in the wood samples are minimized. The volume of these samples is about five times as large as that of the increment core samples. Although this sampling technique might result in a systematic sampling from mainly one compass direction, compression wood usually occurring on the leeside of the tree in areas with prevailing winds from one compass direction, this has no influence on the density level, as the variation of the basic density with the compass direction from a practical point of view is dependant only upon ring width (*Olesen* 1973). After cutting out the sample, a final examination for defects takes place over a lightbox. (It is in order to facilitate this examination that the thickness of the sticks is only 6—8 mm. With thicker sticks, which would not permit the transmission of light, the judgement of defects would be doubtful). The cores (sticks) are then cut up in segments, each containing annual rings of almost equal width, the number of annual rings in each segment being determined by the number of equally wide rings which happens to occur in succession. By this method the basic densities of the narrow and wide annual rings are determined separately, which is very important as the extreme values almost exclusively determine the slope of the regression curve. The values around the mean ring width have almost no effect on the determination of the slope.

The volume of the green wood samples are determined using the water displacement method (*Olesen* 1971), while oven dry weight is determined after 24 hours of drying at 103 C° ± 1°. Both factors are determined with an accuracy of ± 0.1—0.8 per cent, dependent on the size of the test pieces.

The statistical analyses of the two regression curves

$$\mathrm{R} = a + \frac{b}{x + c} \tag{4}$$

$$\mathrm{R} = a + \frac{b}{\sqrt{x + c} + \sqrt{c}} \tag{8}$$

are carried out as linear regression analyses as the regression curves can be transformed to a straight line $\mathrm{R} = a + bx'$, where $x' = 1/(x + c)$, and $x' = 1/(\sqrt{x + c} + \sqrt{c})$ respectively. Thus, linear regression analyses may be applied to the transformed observations, if $\mathrm{f(R)}$ is normally distributed with the mean value

$$M\left\{\mathrm{f(R)} \mid \mathrm{g}(x)\right\} = a + b(\mathrm{g}(x) - \overline{\mathrm{g}(x)})$$

and the variance

$$V\left\{\mathrm{f(R)} \mid \mathrm{g}(x)\right\} = \sigma^2.$$

In the analysis, each segment is given a weight equal to the number of annual rings in the segment. The equations are then solved with respect to R for varying values of c in the interval $0 \leq c \leq 11.0$. A computer programme using the theory of least squares has been developed by Lic. agro. P. Brun Madsen. The programme determines and selects the value of c giving the smallest variance about the regression curve, and thus the equation which best fits the observations.

4. EXAMPLE

The following example is given in order to illustrate the differences between an ordinary linear regression analysis and an analysis of the regression curve $R = a + b/(x + c)$ after its transformation to a straight line. The result of varying c-values will also be demonstrated.

The data used in the example originate from an analysis of the basic density of 15 plus tree candidates. The juvenile wood is excluded and only ring numbers greater than 15 form part of the analysis. The 128 observations are plotted in Figure 7 with one linear and two curvelinear regression lines.

Figure 7 shows that the graph which best fits the observations is a curve in which basic density decreases with increasing ring width. This also is

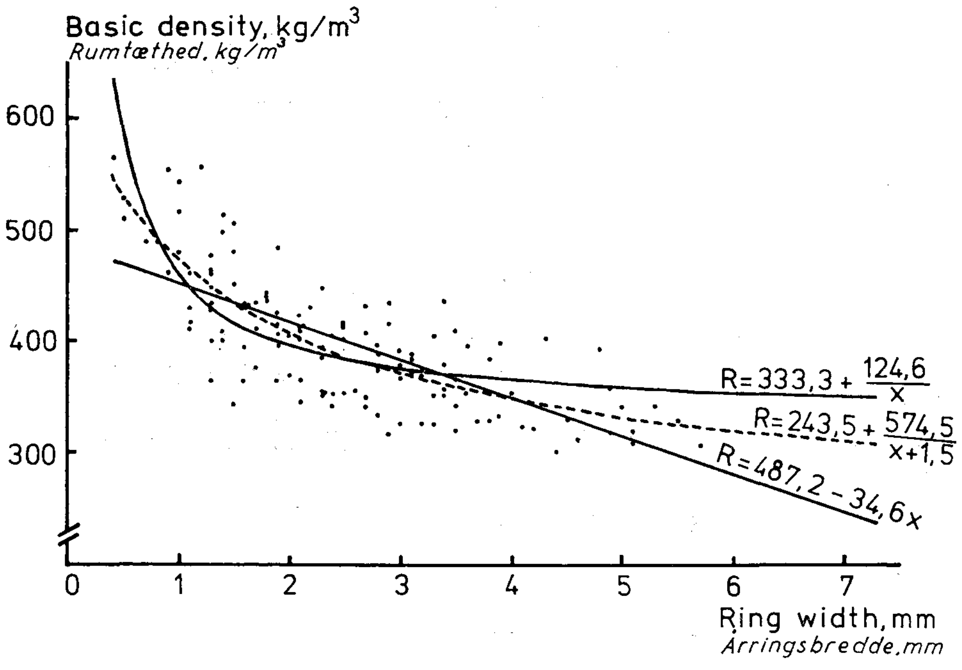


Fig. 7. The interrelation between basic density and ring width for 15 plus tree candidates. A comparison of three regression equations.

Fig. 7. Sammenhængen mellem rumtæthed og årringsbredde hos 15 plustræer. En sammenligning mellem tre regressionsligninger.

in agreement with previous experiences. The result of the regression analyses of the straight line $R = a + bx$ and the hyperbola $R = a + b/(x + c)$ is shown in Table 1 for varying values of c.

Table 1. Comparison of regression analyses.

Equation	a	b	s	s_a	s_b	r
$R = a+bx$	487.2	—34.6	40.5	3.58	2.82	—0.738
$R = a+b/x$	333.3	124.6	40.2	3.55	10.0	0.742
$R = a+b/(x+1)$	267.4	412.8	36.3	3.20	27.9	0.797
$R = a+b/(x+1.5)$	243.5	574.5	36.2	3.20	38.8	0.797
$R = a+b/(x+2)$	221.1	752.4	36.4	3.21	51.1	0.795

The correlation coefficient of —0.738 for the linear regression analysis is unusually high, and the difference between the correlation coefficients from regression analyses using the functions $R = a + bx$ and $R = a + b/(x + c)$ will usually be greater. This will be discussed in more detail later.

5. RESULTS AND DISCUSSION

In the following, results from two investigations carried out at Ålholm and Christianssaede forests are used in the analysis of the two derived equations (4) and (8), cf. p. 347. The standard deviation, s, the coefficient of correlation, r, and the parameter, c, from each of these regressions, are given in Table 2 for the best fits of c.

The correlation coefficients in Table 2 are higher than those found by other authors in similar investigations, i.e. investigations including several trees, and excluding the juvenile wood. For example, Bernhart (1964) and Hakkila (1968) found correlation coefficients of —0.68 and —0.65 respectively applying linear regression analysis. As the square of the correlation coefficients, r^2, may be described as that fraction of the total variance of R which is determined by, or calculable from, the value of x, about 45 % of the variance of R can be attributed to the variation in ring width in these investigations. In the investigations referred to in Tables 1 and 2, about 65 % of the variance of R can be attributed to variation in ring width. Thus, an appreciably better analysis is obtained by combining the technique described in section 3 with the use of the hypothesis, $R = a+b/(x+c)$, which is in accordance with our theoretical and practical knowledge of the behaviour of the basic density with varying ring width.

Table 2. Comparison of equation (4) and (8).

Plot No.	Nos. of segments	(4) $R = a+b/(x+c)$ s	r	c	(8) $R = a+b/(\sqrt{x+c} + \sqrt{c})$ s	r	c
Ålholm, 15 trees per plot:							
1	95	30.0	0.75	3.2	30.0	0.75	1.6
2	68	29.5	0.83	0.8	29.3	0.83	0.0
4	74	28.7	0.74	9.6	28.7	0.74	4.0
5	46	38.0	0.80	1.6	38.6	0.79	1.6
6	47	41.6	0.81	8.0	41.6	0.81	3.2
7	87	37.6	0.61	11.0	37.6	0.61	4.8
9	83	21.9	0.83	3.2	22.0	0.83	1.6
10	79	34.1	0.65	4.8	34.1	0.65	1.6
All plots	579	34.2	0.87	2.0	35.1	0.86	1.6
Christianssaede, 10 trees per plot:							
1+9	77	40.9	0.71	11.0	40.8	0.71	11.0
2+6	81	33.6	0.65	11.0	33.5	0.65	11.0
4+8	67	31.5	0.86	3.2	31.6	0.86	1.6
3	32	34.5	0.88	1.6	35.3	0.87	0.8
5	47	36.8	0.81	6.4	36.8	0.81	3.2
10	39	31.7	0.80	11.0	31.6	0.80	11.0
11	25	41.5	0.71	0.1	41.5	0.71	0.0
All plots	368	38.3	0.76	3.3	38.4	0.76	1.6

From Table 2 it can be seen that the standard deviation for the two equations differs little within plots. On an average, equation (4) is better than (8), but the difference is so small that from a practical point of view the equations seem to be equally good. However, the use of the second hypothesis, $R = a+b/(\sqrt{x+c} + \sqrt{c})$, in most cases leads to a negative value of a for the best fit of c. This is not possible as a theoretically represents the basic density of the earlywood. Furthermore, this equation is more complicated than the first equation. Thus, from a theoretical point of view the use of the first hypothesis

$$\boxed{R = a + \frac{b}{x+c}}$$

leads to the best result, and as this hypothesis also fulfills our requirement to simplicity, this hypothesis is preferred.

The behaviour of this hyperbola is in all respects in accordance with our experience, i.e. the basic density decreases with increasing ring width, with decreasing rapidity, so that it resembles a hyperbola with a horizontal asymptote. Thus, both from a theoretical and an empirical point of view the hyperbola, $R = a+b/(x+c)$ satisfies our knowledge and thereby reflects — at least partly — a causal relationship between basic density and ring width. In the following this regression equation will be dealt with in more detail.

The parameter a determines the level of the curve, as $y = a$ is horizontal asymptote. Thus, a change in a with b and c constant will lead to a displacement of the curve parallel to the y-axis.

The parameter b determines the curvature of the hyperbola, so that increasing b-values with a and c constant lead to increasing curvature. Likewise a change in c, with a and b constant, leads to a displacement of the curve parallel to the x-axis. Although theoretically, $x = -c$ is the vertical asymptote to the curve, the ring width will always be greater than zero, so that the asymptote does not exist in the defined range.

Similarly c should always theoretically be greater than zero as $R \to \infty$ for $x \to 0$ is not possible. However, when material is limited to few samples, the author has often found the best fit with $c = 0$, although when more

Table 3. Comparison of parameters for varying c-values with $R = a+b/(x+c)$.

Material	c	s	a	b
Ålholm, all plots	0.0	45.52	351.8	56.1
	0.5	36.96	305.5	195.5
579 segments from	1.0	34.87	271.5	351.1
	1.5	34.25	241.9	526.0
120 trees	2.0*	34.16	214.7	721.1
	2.5	34.27	189.0	936.6
	3.0	34.46	164.4	1,172.6
	3.5	34.68	140.4	1,429.2
	4.0	34.91	117.0	1,706.5
	8.0	36.37	—60.6	4,667.7
Christianssaede	0.0	46.17	380.1	50.2
	0.5	40.77	322.8	175.4
all plots	1.0	39.31	281.8	325.1
	1.5	38.73	245.8	502.7
368 segments from	2.0	38.47	212.4	708.4
	2.5	38.36	180.4	941.9
100 trees	3.0	38.32	194.4	1,203.3
	3.2*	38.32	137.2	1,315.5
	3.5	38.32	119.1	1,492.2
	4.0	38.34	89.3	1,808.8
	8.0	38.65	—139.6	5,327.3

* best fit.

samples are available, so that random errors play an insignificant role, c has always been greater than zero.

The effect of varying c-values on the standard deviation, s, and on the parameters a, and b, is shown in Table 3.

It can be seen that a variation in c within relatively wide limits has little effect on s, but if $c \rightarrow 0$, the effect on s becomes of practical importance. A variation in c has, however, a great effect on the parameters a and b so that it is not possible to use the estimated value of a as an estimate of the basic density of the earlywood, as the best fit of c will always be subject to some error. For example, the results from Christianssaede gives an a-value of 137.2 kg/m^3 for the best fit of c, which is an unacceptably low value of the earlywood density. This does not make the regression analysis unsuitable for the purpose of fitting the best curve through the observations, but it indicates that care must be taken with values computed from the equation which lies outside the range of the observations. The a-value is such an example, as it is an estimate of the basic density for an infinitely wide annual ring.

Unacceptable values of a can be avoided by selecting the best fit of c within certain limits of a. For example, if it is known that the basic density of the earlywood varies between 200 and 300 kg/m^3, the equation can be solved within these limits.

With fewer samples, low values of a are often found as the samples may be non representative. Low values of a are also found in materials with a rather narrow range of ring width, which could explain the low a-value in the Christianssaede experiment, where the ring width varied from 0.5—4.5 mm, with one exception of $x = 5.8$ mm. In the Ålholm experiment, on the other hand, the ring width varied from 0.2 mm to 7.1 mm with several ring widths greater than 6 mm. Thus, in order to secure a good estimate of a, the widest annual rings should be kept separate when cutting up the increment cores or sticks. The wider the annual ring, the better will be the estimate of the horizontal asymptote.

The best estimate of the average earlywood density may be obtained by determining the density from approximately 25 to 50 earlywood samples. Stratified sampling techniques should be employed to ensure representative samples from all ring width classes.

In order to compare two or more regression curves, the value of c must be the same for the equations compared. For example, if the two regression curves in table 3 are compared, the value of c could be fixed at 2.6, the mean of the best fits for the two regression curves. The two materials could also be pooled and a common c-value computed. It is of course a drawback that the c-value has to be fixed when the identities of two or more populations are tested. On the other hand, even a relatively great change in c has little effect on the standard deviation, so that the necessary adjustment of c is not considered to be of great importance to the regression analysis.

When the three results in tables 1 and 3 are compared, it is striking to see how similar the parameters a and b are for the same c-values, cf. Table 4.

Table 4. Comparison of the parameters a and b for $c = 2.0$.

Locality and nos. of trees	s	a	b
Tokkekøb, 15 trees	36.4	221.1	752.4
Ålholm, 120 trees	34.2	214.7	721.1
Christianssaede, 100 trees	38.5	212.4	708.4

The striking similarity is found not only for $c = 2.0$, but also for other values of c. It would be interesting to follow the variation of the parameters in future investigations, especially the variation of c, as it would be of great help if c could be fixed for Norway spruce *populations*. The value of c may vary from tree to tree, but it may be a constant for Norway spruce populations.

The above mentioned regression curve is characteristic for Norway spruce. However, earlier investigations show that some Abies, Larix, Pinus, and Pseudotsuga species have a distinctly different curvature, as the density at first increases and attains a maximum for a ring width about 1 mm and then decreases again with increasing ring width (Kollmann 1951), so that it resembles the curve in Figure 8.

If the interrelation between latewood width and ring width is assumed to be a logarithmic function of the type

$$z = u\mathrm{e}^{-c/x} \qquad (11)$$

the graph in Figure 9 is obtained, which is only slightly different from the graph in figure 3. If (11) is substituted in (2) we get

$$\mathrm{R} = \frac{\overline{\mathrm{R}}_{\mathrm{E}}\,(x - u\mathrm{e}^{-c/x}) + \overline{\mathrm{R}}_{\mathrm{L}}u\mathrm{e}^{-c/x}}{x}$$

or

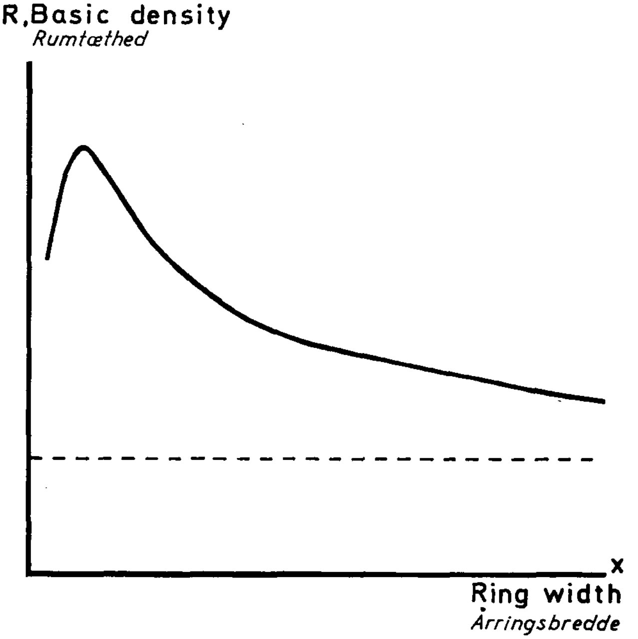


Fig. 8. $R = a + \frac{be^{-c/x}}{x}$

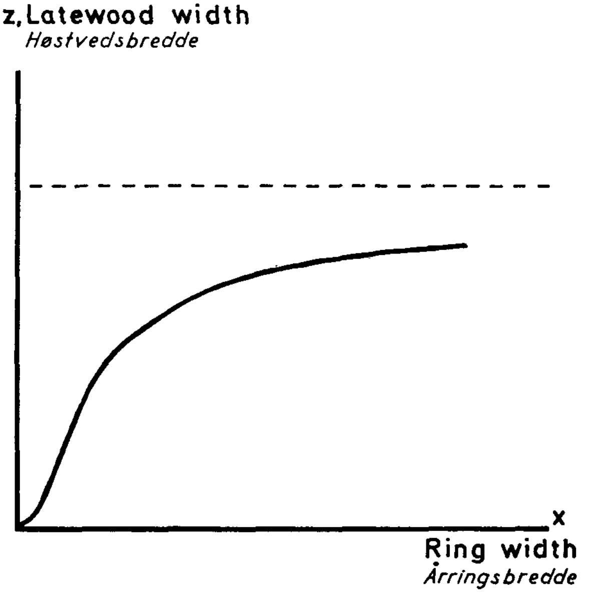


Fig. 9. $z = ue^{-c/x}$

$$R = a + \frac{be^{-c/x}}{x} \tag{12}$$

which gives a graph with a maximum for $x = c$, and with the horizontal asymptote $R = a$, cf. Figure 8.

The form of the regression curve in Figure 8 seems to be in accordance with the earlier findings, summarized by *Kollmann* (l.c.), and might be useful to some research workers. As yet, equation (12) has not been tested using actual data.

ACKNOWLEDGEMENT

In the main, the present paper is a result of investigations carried out while I have been in receipt of a Senior Research Fellowship granted by The Royal Veterinary and Agricultural University, Copenhagen. Assistance has been granted by Statens jordbrugs- og veterinærvidenskabelige Forskningsråd. I am grateful to both institutions for their support.

I also wish to acknowledge the help of *Peter Brun Madsen* in preparing the computer programmes, and of *T. Lynge Madsen* in processing most of the data, and to thank the latter and *Yrsa Andersen* for performing the laboratory work.

SUMMARY

The aim of this study is to derive a model which describes the causal interrelation between basic density and ring width, in accordance with empirical knowledge.

Based on the knowledge of the average basic density of the earlywood and latewood in relation to ring number, and the interrelation between latewood width and ring width, two regression equations are derived of which the following is preferred:

$$R = a + \frac{b}{x+c}, \quad (x > 0)$$

where R = the basic density, x = the ring width, and a, b, and c are three positive constants. The equation represents a hyperbola with the horizontal asymptote $R = a$, and a theoretical, vertical asymptote $x = -c$.

When the above regression equation was tested using samples from 240 Norway spruce, the form of the hyperbola was found to be in accordance with practical experience. The correlation coefficients found were high, about 0.8. Furthermore, the model is easy to use, as it can be transformed to a straight line $R = a + bx'$, where $x' = 1/(x + c)$, so that the theory of linear regression may be applied.

DANSK RESUME

Formålet med nærværende arbejde er at udlede en formel, der beskriver den kausale sammenhæng mellem rumtæthed* og årringsbredde i overensstemmelse med den foreliggende empiriske viden.

Ud fra kendskabet til vårveddets og høstveddets rumtæthedsvariation med årringsnummer (regnet fra marven) og sammenhængen mellem høstvedsbredde og årringsbredde er to regressionsligninger udledt, af hvilke den følgende er foretrukket:

$$R = a + \frac{b}{x+c}, \quad (x > 0)$$

hvor R = rumtætheden, x = årringsbredden og a, b og c tre positive konstanter. Ligningen fremstiller en hyperbel med asymptoterne $y = a$ og $x = -c$.

Regressionsligningen er testet på boreprøver fra 240 træer fra Ålholm og Christianssæde skovdistrikter, og de beregnede hyperbler fundet i overensstemmelse med erfaringsmaterialet. De fundne korrelationskoefficienter er relativt høje, omkring 0.8 mod normalt 0.65 ved lineær regressionsanalyse af tilsvarende materialer. Hyperbelfunktionen er desuden nem at arbejde med, idet den kan transformeres til en ret linie $R = a + bx'$, hvor $x' = 1/(x + c)$, hvorefter lineær regressionsanalyse kan anvendes. Der er udviklet et komputerprogram, der opsøger den værdi af c, som giver den mindste spredning omkring regressionslinien. Denne værdi er omkring 2.

LITERATURE

Bernhart, A., 1964: Über die Rohdichte von Fichtenholz. — Holz Roh- u. Werkstoff 22: 215—28.

Bertog, H., 1895: Untersuchungen über den Wuchs und das Holz der Weisstanne und Fichte. — Forstl. Naturwiss. Z. 4: 97—112, 177—216.

Ericson, B., 1960: Studies of the genetical wood density variation in Scots pine and Norway spruce. — Skogsforskn. Inst., Stockh. Rapp. Nr. 4: pp. 55.

Hakkila, P., 1966: Investigations on the basic density of Finnish pine, spruce and birch wood. — Commun. Instituti Forestalis Fenniae 61.5: pp. 98.

Hakkila, P. & O. Uusvaara, 1968: On the basic density of plantation-grown Norway spruce. — Commun Instituti Forestalis Fenniae 66.6: pp. 23.

Johansson, D., 1939: Något om vår- och höstved hos tall och gran och dess inverkan på sulfit- och sulfatmassans egenskaper. — Pappers- och Trävarutidskrift för Finland 21, Nr. 7 A: 54—72.

Klem, G. G., 1934: Undersøkelser av granvirkets kvalitet. — Medd. norske Skogsforsøksv. 5: 199—348.

Klem, G. G., 1957: Kvalitetsundersøkelser av norsk og tysk gran. — Medd. norske Skogsforsøksv. 48: 290—314.

Kollmann, F., 1951: Technologie des Holzes und der Holzwerkstoffe. — 2nd Ed., Vol. 1, Springer-Verlag, Berlin-Göttingen-Heidelberg.

* Kg tørstof pr. m^3 frisk volumen.

Nylinder, P., 1951: Beräkning av höstvedhalt och medelårsringsbredd. — Medd. Skogsforsknings Inst., Stockh. 40 (Nr. 10): 1—40.

Nylinder, P., 1953: Volymviktsvariationer hos planterad gran. — Medd. Skogsforsknings Inst., Stockholm 43 (Nr. 3): pp. 44.

Olesen, P. O., 1971: The water displacement method. — For. Tree Impr., Arbor., Hørsholm No. 3: 3—23.

Olesen, P. O., 1973: The influence of the compass direction on the basic density of Norway spruce and its importance for sampling for estimating the genetic value of plus trees. — For. Tree Impr., Arbor., Hørsholm, No. 6: pp. 58.

Panshin, A. J. and C. de Zeeuw, 1970: Textbook of wood technology. — Vol. 1, McGraw-Hill Book Company, New York.

Trendelenburg, R., 1936: Aufbau und Eigenschaften des Fichtenholzes und anderer Zellstoffhölzer. — Papier-Fabrikant 34: 389, 401, 411, 420, 433.