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GENERAL VOLUME TABLE FOR BEECH IN DENMARK

(ALMINDELIG MASSETABEL FOR BØG I DANMARK)

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§ 1. INTRODUCTION

Investments in forests being long-term investments, mistakes may be made in the dispositions over fairly long periods before the direct economic consequences are discovered, and when this happens, it will often be too late — and at any rate a lengthy and expensive affair — to try to remedy matters. For this reason the measuring of trees was developed rather early for the purpose of following the growth of the individual forests as closely as possible.

When a tree has been felled, it will, with a suitable division into sections, in principle not be difficult to determine its volume with sufficient accuracy. But a tree on the root is a different problem, and this is what we are faced with in practice when estimating or studying the increment of a stand. For the determination of the volume of a standing tree, the diameter of the tree (measured at the customary height above the ground) and its height in the first place present themselves. From these figures, however, the volume of the tree cannot be directly calculated; it must be taken into consideration that the tree cannot be regarded as a cylinder, but tapers towards the top (and widens a little at its basal part, below the height of measurement). On this account the volume of the cylinder (the product of the height and the basal area) must be reduced to a certain extent, i. e. it must be multiplied by a figure less than 1, and in this way we arrive at the important and generally applied concept of *form factor,* the term now current for this reduction.

The term form factor can be traced back to the term "Reduktionszahl" (reduction figure) introduced by Joh. Christ. Paulsen in his paper "Über die richtigste Art der Berechnung des Zuwachses an ganzen Holzbeständen" (1800). The term "Formzahl" (form factor) was first used by König (1813) and has since then become firmly established; but the term "reduction figure" is in so far more adequate, as we are here actually concerned with a reduction, not with a determination of the form.

The original reason for introducing the term form factor (reduction figure) must have been the hope that the form factor was — or might with sufficient approximation be regarded as — a constant characteristic of each species of tree. The form factor might then be determined once and for all by accurate measurement of a comparatively small number of trees, and in all subsequent determinations of volume the product of height and basal area merely had to be multiplied by the known form factor.

However, conditions are not so simple as this, and attempts have therefore been made to establish a more convenient theory by assuming that the form factor is variable and dependent on certain measured — or measurable — quantities, and the most natural would then be to regard the form factor as a function of the height and the diameter, which, at any rate, have to be measured. The practical consequence of this was the preparation of tables with entry by means of height and diameter. The first tables of this kind were the Bavarian volume tables from 1846. The largest and now most frequently employed tables were prepared after the foundation of the "Verein deutscher forstlicher Versuchsanstalten" (1872); here the volume tables prepared by Grundner and Schwappach should especially be mentioned. Such tables give the very volume desired, but in the calculation of the tables the form factors were used, because it was found easier to make an adjustment of the almost equally large form factors than of the highly varying volume figures.

It turned out, however, that even in the form mentioned here the theory did not quite come up to what was expected of it, and this gradually became all the more obvious, as the demand for accuracy increased. Various attempts were then made to improve the method for the determination of the form factor and the volume by including other measurable quantities. As one of the first attempts in this direction we may mention Pressler's "Richthöhemethode" (1855), according to which the form factor is determined by fixing that place on the trunk (Richthöhe) where the diameter equals half the diameter at a height of 1.3 m (international height of measurement). This method has not, presumably, been much employed. More interest attaches to the measurement of a form quotient, i. e. the ratio between the diameters at two specified points of the trunk. Such a form quotient, determined as the ratio between the diameter at the middle of the trunk (half the height of the tree) and the diameter at 1.3 m,

is included in Schiffel's tables (Norway spruce 1899, larch 1903, pine 1907). Tor Jonson uses in his tables (pine and Norway spruce, 1910—11) the "absolute form quotient", i. e. the ratio between the diameter at the middle of the piece of trunk above 1.3 m and the diameter at 1.3 m. The crown ratio has also been used; from quite recent years we may mention Nässlund, who has both prepared tables with entry by means of height and diameter alone (pine, Norway spruce, birch) and tables with entry by height, diameter, and crown ratio (pine, Norway spruce). Related to the measurement of the crown ratio is Tor Jonson's determination of the "form point", i. e. the point where the wind attacks the crown. Furthermore the age has been employed, certain tables having a division into age classes; this applies e. g. to some of Grundner's and Schwappach's tables.

In our view, at any rate as far as the deciduous trees are concerned, no great progress has been made by the above-mentioned methods. The reason is presumably that the form factor and, accordingly, the volume further depend on a number of local conditions as, for instance, race, treatment, soil, and climate, in a way which has not been taken into account above, and which it will be very expensive to take into account to any great extent for each single stand.

As a consequence of what has been stated above there might be reason to submit the volume determination of a stand to a renewed careful scrutiny. The present paper is the result of a cooperation with the Danish Forest Experiment Station, and is based on a mathematical treatment of an experimental material which the Station has placed at our disposal. Mr. H. A. Henriksen, head of the measuring department of the Station, let us benefit by his expert knowlidge in the field of forestry by giving us valuable assistance throughout our work. We think that in the very varied material we have found certain general features which render possible a decisive improvement of the results. Our considerations have resulted in the establishment, on the basis of the material available, of a volume table with entry by means of diameter and height, which may be used for the determination of the total volume of a stand in the following two fundamentally different ways:

1) All diameters and heights are measured. The volume of each individual tree is found by means of the table. In this way

the total volume is determined with a standard error of ca. 5 per cent.

2) All the diameters and heights are measured, and the individual volumes are determined as under 1). In addition direct determination of volume is made on a few trees which give a natural expression of the variation in the stand (thus, for instance, not "mean trees" or trees of poor quality), and the values thus found are compared with the table values. On this basis a correction factor is computed, which may be used for the stand in question, and is an expression of the individual features of the place. If the total volume found by means of the table is corrected in agreement with this, the accuracy is increased, and the risk of a systematic error is considerably reduced. If, for instance, such a direct determination of volume is made for nine trees, a reduction of the standard error to 2.3 per cent may be counted upon. If the determination is made for four trees only, the standard error will be 3.5 per cent. (cf. § 8).

If not all the heights are measured, but by some adjustment method approximate values are found for a number of heights on the basis of measured diameters, the above-mentioned accuracy will, of course, be somewhat reduced. If not all the diameters are measured, either, but only a representative section of the forest instead of the whole is considered, the uncertainty of the result will be further increased. These latter problems will not be dealt with in the present paper.

§ 2. DIRECT CONTEMPLATION OF THE EXPERIMENTAL MATERIAL

The material used is derived from the Danish beech stands, and falls naturally into three groups. Group I consists of measurements carried out by the Forest Experimental Station in 17 stands, distributed over the various parts of the country and consisting of from 10 to 38 trees[1]), while groups II and III com-

[1]) In Report 149 mentioned as the "main material". As to the character of this material and the method by which it was measured, see further Report 149, pp. 4—9 and 20—24. Either all the trees of a stand were used, or the employed trees were selected in such a way as to give a fairly complete representation of the stand spectrum. Slightly in conflict with this principle, a part of the sample tree material from sample plot M was included for representative reasons. Even though the 31 trees measured in this sample plot are not of precisely the same age, it appears that the results are not noticeably influenced by the inclusion or omission of this material.

prise measurements from a number of stands, 11 and 8 respectively, from the Bregentved and Sorø forest districts, consisting of from 6 to 15 trees[2]).

Below the forestry denotions are given for all employed stands, as well as the abbreviations used hereafter.

Forestry denotion (in danish)	*abbreviation*
Group I.	
Prøveflade (sample plot) M	I 1
Prøveflade (— —) S	I 2
Esrum distrikt, afdeling 281	I 3
Esrum distrikt, afdeling 307	I 4
Frederiksdal distrikt, Storskoven	I 5
Aastrup distrikt, afdeling 61	I 6
Bonderup distrikt, Merløse afdeling 17	I 7
Bonderup distrikt, Merløse afdeling 19	I 8
Bonderup distrikt, Merløse afdeling 21	I 9
Giesegaard distrikt, Maglebjerg skov	I 10
2. Sorø distrikt, Lille Bøgeskov, afd. 82 B	I 11
Petersgaard distrikt, Stensby skov	I 12
Brahetrolleborg distrikt, Storskoven	I 13
Aarhus distrikt, Skaade skov, afdeling 95, 1	I 14
Aarhus distrikt, Skaade skov, afdeling 95, 2	I 15
Boller distrikt, Randskoven, afdeling 6—7	I 16
Stensballe distrikt, Kærskoven, afdeling 29	I 17
Group II (Bregentved skovdistrikt)	
Boholte skov, afdeling V 16	II 1
Stubbekrogen, afdeling XXV 1	II 2
Ganneskov, afdeling XXII 69	II 3
Børsted skov, afdeling XVI 20	II 4
Ganneskov, afdeling XXII 36	II 5
Karise Hestehave, afdeling XXI 3	II 6
Boelskov, afdeling XXIII 33	II 7
Boelskov, afdeling XXIII 10	II 8
Grevindeskoven, afdeling VI 96	II 9
Haslev Orned, afdeling I 47	II 10
Boelskov, afdeling XXIII 59 a	II 11

[2]) The measurements of groups II and III were carried out as "sample tree measurements" collected by the district administration as parts of more comprehensive valuation work.

Forestry denotion (in danish)	*abbreviation*
Group III (Sorø skovdistrikt)	
Søskoven, afdeling 17	III 1
Søskoven, afdeling 10 A	III 2
Vindelbro skov, afdeling 30 B	III 3
Alsted skov, afdeling 1 A	III 4
Alsted skov, afdeling 25 B	III 5
Alsted skov, afdeling 50 B	III 6
Vesterskov, afdeling 69 A	III 7
Vesterskov, afdeling 101 A	III 8

For each tree i.a. the following measurements were available:

1) The trunk diameter, d_0, measured at a height of 1.3 m above the ground.
2) The trunk diameters $d_{0.1}$, $d_{0.2}$, . . . , $d_{0.9}$, measured at heights above 1.3 m, which constitute $^1/_{10}$, $^2/_{10}$, ..., $^9/_{10}$, respectively, of that part of the trunk which is above the height of 1.3 m.
3) The height of the tree h.
4) The form factor of the trunk f.
5) The branch volume G.
6) The total volume T.

The distinction between trunk volume and branch volume arises automatically with the measuring technique employed, but is only of little interest in the treatment of the material, this distinction being very ill-defined in nature.

In order to gain a thorough insight into such a material it will be natural, as a beginning, to compare the stated quantities in pairs; this is done most clearly by a graphic representation. A number of diagrams were drawn; as an example we will mention in more detail those in which the total volume T is plotted against the height h and the diameter d_0, respectively. In figs. 1—2 this is shown for the stand I 15, and in figs. 3—4 analogously for the stand I 7. The total volume T increases everywhere with increasing h and d_0, and the absolute differences between the trees are greater for the large trees than for the small ones. All this is well known and quite natural.

Now, experience has shown that as regards growth problems it is most frequently the case that the logarithms of the quantities under consideration vary according to simpler laws than the

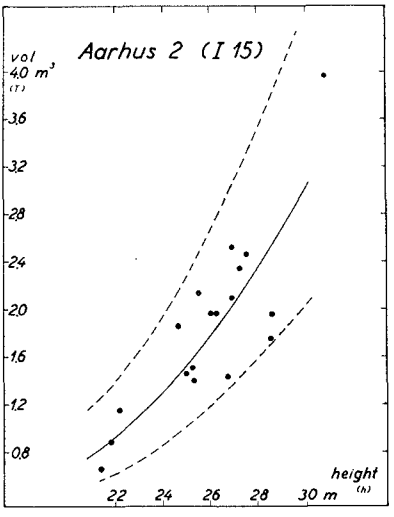

Fig. 1.

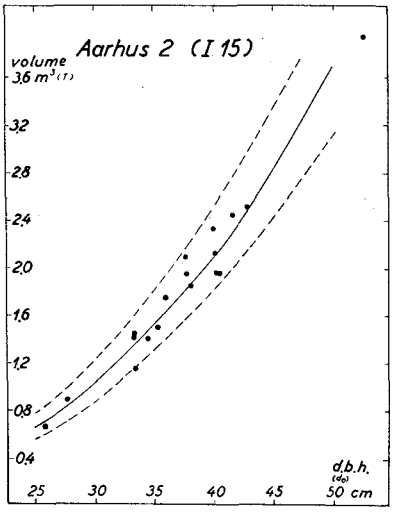

Fig. 2.

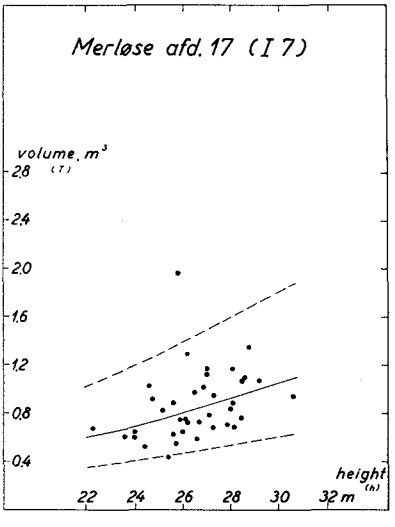

Fig. 3.

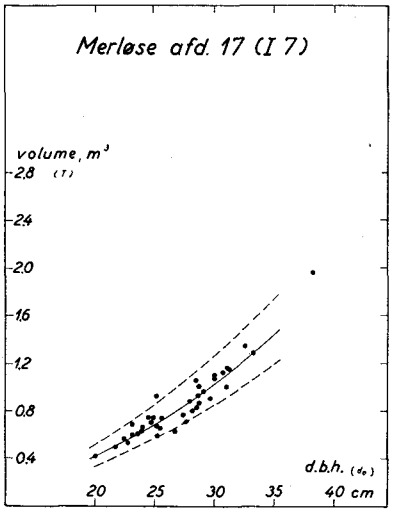

Fig. 4.

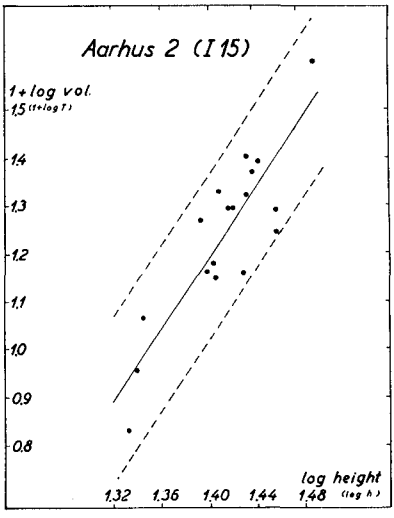

Fig. 5.

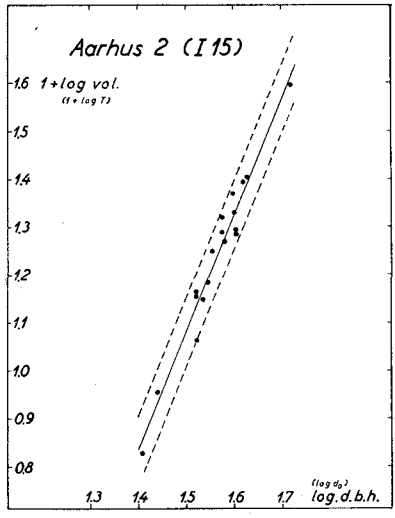

Fig. 6.

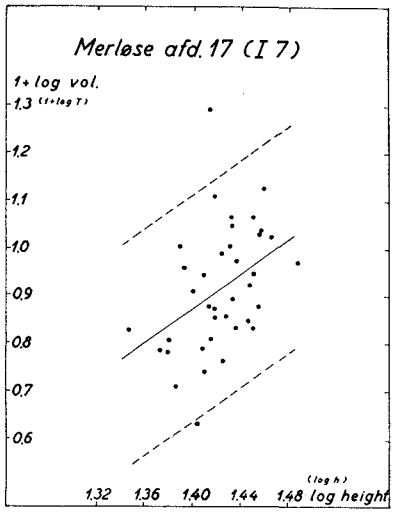

Fig. 7.

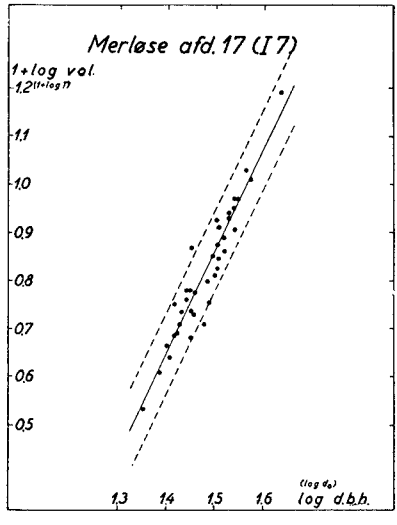

Fig. 8.

quantities themselves. In the present case, if we take the logarithms of the measurements and plot them against each other, the pictures will assume a different character (see figs. 5—8). (The same result may be obtained by using the original measurements in connection with double-logarithmic paper.) From this it appears that the pictures, which were previously curved, have now been straightened out, and the former funnel-shaped appearance has changed in such a way that the figure has almost a constant breadth. The connections between the logarithmic measurements are accordingly linear; further, the variation of log T is independent of the size of log h and log d_0. This latter fact may also be expressed by saying that the percentage variation of T is independent of the size of h and d_0.

According to figs. 5 and 7 we then have with approximation

$$\log T = \alpha + \beta \log h, \tag{1}$$

where α and β are constants, and analogously for log T's dependence on log d_0 (figs. 6 and 8). The approximations, however, are not equally good, thus, for instance, the deviations from the straight line will be seen to be greater in figs. 5 and 7 than in figs. 6 and 8. The diameters are accordingly more suitable than the heights for a determination of the volume of the trees.

Now, it would be natural to expect that the approximation would be materially improved if we proceed to consider T's simultaneous dependence on two or more of the aforementioned quantities and assume log T to be a linear expression in the logarithms of these quantities. If, for instance, we are concerned with T's dependence on h and d_0, we will put

$$\log T = \alpha + \beta \log h + \gamma \log d_0, \tag{2}$$

where α, β and γ are constants (and α and β have not, probably, the same values as in (1)). Formula (2) is equivalent to

$$T = 10^{\alpha} h^{\beta} d^{\gamma}, \tag{3}$$

but (2) is more suitable for practical calculations.

Above we have dealt with the dependence of the *total volume* on other factors, not with the *form factor*'s dependence on these. At to this it may in the first place be said that it is the volume which it is desired to determine, whereas the form factor is merely an auxiliary quantity. It is decisive, however, that if we pass

on to logarithmic measurements, the justification of the use of form factor vanishes entirely. For from

$$T = \frac{\pi}{4} d_0^2 \, h \, F, \tag{4}$$

where F is the total form factor, we get

$$\log T = \log \frac{\pi}{4} + 2 \log d_0 + \log h + \log F, \tag{5}$$

which combined with (2) gives

$$\log F = (\alpha - \log \frac{\pi}{4}) + (\beta - 1) \log h + (\gamma - 2) \log d_0. \tag{6}$$

Thus, if T satisfies an equation of type (2), the same applies to F, and conversely; there is therefore no reason to abandon the quantity T which actually is to be determined.

It will further be seen that the original theory with a constant F corresponds to the fixing, beforehand, in (3) and (2), of $\beta = 1$ and $\gamma = 2$. It is obvious that when β and γ are set free and it is attempted to determine them on the basis of the observations, formula (2) will lend itself considerably more adaptable to these.

§ 3. OUTLINE OF THE FURTHER PROCEDURE

Through the considerations set forth above, including the accompanying figures, it is made clear that log T may approximatively be expressed linearly by the logarithms of various other measurable quantities, as for instance h and d_0. We will now especially consider the connection between T and h, expressed by § 2, (1). Geometrically we may say that the "points of observation" group around a certain line, the regression line. As a measure of the goodness of the approximation we have the so-called standard deviation (mean error), which is a concentrated expression of the vertical deviations of the points from the line and accordingly also of the differences between the actual total volumes and those that would be obtained by measuring the heights and then employing the regression line.

Similarly, we may give a geometric interpretation of § 2, (2) by adopting a system of coordinates in space. The points are distributed around the regression plane determined by § 2, (2), and, as above, we may speak of a standard deviation as an expression of the deviations of the points from the plane and accordingly of the differences between the actual total volumes and those that may be calculated by measuring h and d_0 and using the plane.

If we look at T's simultaneous dependence on three or more quantities, the geometric interpretation fails; but the corresponding apparatus of calculation functions equally well, as will be seen later.

Let us consider more closely T's simultaneous dependence on h and d_0, that is § 2, (2). A calculative treatment of this must aim at determining, for each stand considered, a regression plane and a corresponding standard deviation. Imagining that this has been done, we shall find a total of $17 + 11 + 8 = 36$ regression planes and just as many standard deviations. If the method is to be of practical use, the results found must have certain features in common.

In the first place, it is desirable that the standard deviations found should be so nearly equal that the differences can be explained in a natural way as due to chance. This, indeed, turns out, in the main, though not entirely, to be the case. Keeping in this preliminary description to essentials, we will proceed on the hypothesis that the true standard deviations (for which those found are approximations) are equal, that is, that there exists a common measure of accuracy which is valid for all stands. If this did not hold good — at any rate approximately — the mathematical theory would become materially complicated.

As regards the regression planes found, it would, of course, be best if they all coincided, apart from minor deviations which might be regarded as random. On closer examination this proves not to be the case, and consequently it will not be possible to prepare volume tables where by means of heights and diameters we can find the total volume of a stand, only encumbered with random errors, the percentage amount of which is negligible when the number of trees is large.

Now, if this cannot be achieved, it might be imagined that the planes were mutually parallel except for minor random deviations in direction. Something of the kind actually proves to be the case, and it is this which renders possible the fairly simple conclusion which will be the result of the present paper. Thus, to two different stands there will generally correspond two parallel regression planes, situated at different heights or, as we shall say, at different "levels". This means that if we have two trees each from its particular stand, but with a common h and a common d_0, the tree from the stand with the highest level will always

(except for minor random differences) contain a certain higher percentage of volume than the other, and this excess percentage will be independent of the size of h and d_0.

In this way the situation already suggested in § 1 will arise: We prepare once for all, on the basis of a large material, a table with entry from the height h and the diameter d_0 for the determination of the total volume, corresponding to a certain mean level. If this table is used generally, after measurement of heights and diameters, one runs the risk of committing a systematic error corresponding to the difference of level. The resulting standard deviation may, as previously mentioned, be estimated at 5 per cent. In unfavourable cases one may therefore, by such a use of the table, risk an error of 10 per cent or a little more. If this is consistent with the degree of accuracy required, the method is applicable. In the opposite case the said measurements may be supplemented by direct determinations of volume for some few trees; in this way we may find an approximate value for the difference in level and in the main eliminate the systematic error (cf. § 8).

It should be added that in the above it has not been considered whether the differences in level ascertained in the material are due to topographical conditions or to errors in the mode of selection. For such an investigation the available material was unsuitable.

In the following sections the considerations and calculations which have led to the above-mentioned results will be dealt with in more detail.

§ 4. COMPARISON OF TWO STANDS

We will consider the above-mentioned equation § 2, (2), viz.

$$\log T = \alpha + \beta \log h + \gamma \log d_0, \tag{1}$$

and give the calculations and results for the aforementioned stand I 15. For the other stand, I 7, we will merely give the results, found by the same method, and the two sets of results will then be subjected to a critical comparison. We will commence by offering some explanatory remarks on the mathematical symbols and formulas employed[1]).

[1]) See A. Hald, Statistiske Metoder, Kbhvn. 1948, p. 500 ff. and the accompanying tables. The designations, however, are not quite the same.

For the sake of brevity we put $\log h = x$, $\log d_0 = y$, and $\log T = z$, so that (1) becomes

$$z = \alpha + \beta x + \gamma y. \tag{2}$$

The number of trees is called n. A bar over a letter denotes the forming of an average (mean), thus e. g.

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n} \Sigma x_i.$$

It proves convenient to transform (2) into the form

$$z = \alpha + \beta (x - \bar{x}) + \gamma (y - \bar{y}). \tag{3}$$

The coefficients β and γ (but not α) here have the same meaning as in (2).

For certain frequently occurring sums of squares and of products we introduce abbreviations, as for instance

$$SK_x = \Sigma (x_i - \bar{x})^2, \qquad SP_{xy} = \Sigma (x_i - \bar{x})(y_i - \bar{y}).$$ [1]

These quantities are computed from the formulas

$$SK_x = \Sigma x_i^2 - \frac{1}{n} (\Sigma x_i)^2, \; SP_{xy} = \Sigma x_i y_i - \frac{1}{n} \Sigma x_i \Sigma y_i.$$

From the available numerical material approximate values ("estimates") are sought for the theoretical quantities α, β, and γ introduced in (3) and for the standard deviation σ from the regression plane[2]).

For α we have the estimate $\bar{z}$; the estimates b and c for β and γ are found from the equations

$$\begin{cases} b\, SK_x + c\, SP_{xy} = SP_{xz} \\ b\, SP_{xy} + c\, SK_y = SP_{yz}. \end{cases} \tag{4}$$

Then we form

$$q^2 = SK_z - b\, SP_{xz} - c\, SP_{yz}, \tag{5}$$

which has $f = n - 3$ "degrees of freedom", and the estimate s for the standard deviation σ is then determined by

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[1]) By Hald termed SAK_x and SAP_{xy}.
[2]) See Hald, p. 500 ff.

$$s^2 = \frac{q^2}{f} = \frac{q^2}{n-3}. \quad (6)$$ [1])

Table 1.
Calculation of means, square sums, and product sums for the stand I 15.

tree no:	m h	cm d_0	m^3 T	x log h	y log d_0	z log T
1	26.8	33.3	1.463	1.428	1.522	0.157
2	27.3	39.9	2.343	1.463	1.601	0.370
3	27.0	37.6	2.097	1.431	1.575	0.322
4	27.6	41.5	2.464	1.441	1.618	0.392
5	25.4	34.4	1.409	1.405	1.537	0.149
6	28.6	35.9	1.764	1.456	1.555	0.246
7	24.7	38.0	1.857	1.393	1.580	0.269
8	25.6	40.0	2.135	1.408	1.602	0.329
9	21.9	27.6	0.901	1.340	1.441	— 0.045
10	28.6	37.7	1.957	1.456	1.576	0.292
11	27.0	42.7	2.521	1.431	1.630	0.402
12	30.7	52.5	3.957	1.487	1.720	0.597
13	22.2	33.4	1.163	1.346	1.524	0.066
14	25.0	33.3	1.451	1.398	1.522	0.162
15	25.3	35.2	1.512	1.403	1.547	0.180
16	26.3	40.2	1.974	1.420	1.604	0.295
17	21.6	25.7	0.674	1.334	1.410	— 0.171
18	26.1	40.3	1.969	1.417	1.605	0.294
			sum	25.430	28.169	4.306

$\bar{x} = 1.4128$	$\bar{y} = 1.5649$	$\bar{z} = 0.2392$
$SK_x = 0.028297$	$SK_y = 0.083279$	$SK_z = 0.523129$
$SP_{xy} = 0.039571$	$SP_{xz} = 0.106308$	$SP_{yz} = 0.204695$

The subscripts 1 and 2 are used below to indicate that the quantities have reference to I 15 and I 7, respectively.

[1]) The method here described for the determination of b, c, q^2 (and accordingly s^2) is easy to explain and very useful in many cases; it is intended to serve as a basis for the preliminary description of the calculations given in this and the following two sections. In the calculations as actually carried out, however, another method was used, which presents various advantages when it is desired to vary the number of variables included. This method is described in § 7.

According to table 1, which contains the calculations for I 15,

$$\bar{z}_1 = 0{,}2392.$$

The equations (4) for the determination of b_1 and c_1 become

$$0.028297\ b_1 + 0.039571\ c_1 = 0.106308$$
$$0.039571\ b_1 + 0.083279\ c_1 = 0.204695.$$

From this we obtain

$$b_1 = 0.952643,\ c_1 = 2.005281.$$

From (5) we find $q_1^2 = 0.011384$, and as $f_1 = 18-3 = 15$, we get from (6)

$$s_1^2 = 0.000759,\ s_1 = 0.0275.$$

As to I 7 we proceed in a quite similar way. Here we get

$$\bar{z}_2 = -0.0823,$$

and the equations (4) become

$$0.031012\ b_2 + 0.013142\ c_2 = 0.057760$$
$$0.013142\ b_2 + 0.128624\ c_2 = 0.272993.$$

From these we find

$$b_2 = 1.006674,\ c_2 = 2.019553.$$

Further we get $q_2^2 = 0.039604$, and as $f_2 = 38-3 = 35$,

$$s_2^2 = 0.001132,\quad s_2 = 0.0336.$$

These two sets of results will now be compared. According to the statements in § 3, the comparison falls into three parts:

A) An investigation as to whether the difference between s_1 and s_2 may be of random nature[1]). We form

$$v^2 = \frac{s_2^2}{s_1^2} = \frac{0.001132}{0.000759} = 1.49.$$

From a table of the v^2-distribution it will be seen that with 35 and 15 degrees of freedom for numerator and denominator, $v^2_{0.975} = 2.61$; this corresponds to a two-sided 95 % limit, with 2½ % cut off on either side. As the v^2 found is even substantially

[1]) See Hald, pp. 277—278 and table VII. Tables of a similar kind are also found i. a. in R. A. Fisher, Statistical Methods for Research Workers; a table of $v^2_{0.975}$, however, is not found in the latter work.

smaller than 2.61, s_1 and s_2 may quite well be regarded as two estimates of a common standard deviation. These are combined in the following way: We form

$$(7) \qquad q^2 = q_1^2 + q_2^2 = 0.050988, \quad f = f_1 + f_2 = 50.$$

The value s of the common standard deviation is then obtained from

$$(8) \qquad s^2 = \frac{q^2}{f} = 0.001020, \quad s = 0.0319.$$

B) An investigation as to whether the deviation in direction of the two planes may be random, that is, whether the difference between the pairs of figures (b_1, c_1) and (b_2, c_2) may be of a random origin[1]). We will tentatively assume this, and can then determine the common set (b, c) from the equations

$$(9) \qquad \begin{cases} b \Sigma SK_x + c \Sigma SP_{xy} = \Sigma SP_{xz} \\ b \Sigma SP_{xy} + c \Sigma SK_y = \Sigma SP_{yz}, \end{cases}$$

where e. g. ΣSK_x indicates the sum of the values for SK_x, corresponding to the two stands under consideration, and analogously for the other expressions.

We then need the quantity

$$(10) \qquad q'^2 = \Sigma (b_r - b)^2 SK_x + 2 \Sigma (b_r - b)(c_r - c) SP_{xy} + \Sigma (c_r - c)^2 SK_y, \qquad (r = 1,2).$$

For the computation of q'^2 the easiest way is to use a transformation, so that first we find

$$(11) \qquad \bar{q}^2 = \Sigma SK_z - b \Sigma SP_{xz} - c \Sigma SP_{yz}$$

and then make use of the fact that

$$(12) \qquad q'^2 = q^2 - \bar{q}^2.$$

As q'^2 has two degrees of freedom, s' , determined by

$$(13) \qquad s'^2 = \frac{1}{2} q'^2,$$

will be a new value for the standard deviation, independent of s in (8) and based on the hypothesis of parallel regression planes. The test of the correctness of the hypothesis therefore consists in a comparison of s and s'.

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[1]) Cf. Hald, pp. 447—48, where, however, it is a comparison of two regression *lines* that is made.

In the present numerical example the equations (9) become

$$0.059309\ b + 0.052713\ c = 0.164068$$
$$0.052713\ b + 0.211903\ c = 0.477688,$$

which have the solutions

$$b = 0.979267,\ c = 2.010675.$$

From (11) we then fin $\bar{q}^2 = 0.051060$, and then by (12) and (13)

$$q'^2 = 0.000072, \qquad s'^2 = 0.000036.$$

An erroneous hypothesis will, according to (10), give too high a value for s'^2, and since here $s'^2 < s^2$, the hypothesis can be maintained. As a consequence hereof the values for b and c found above may be used in the regression planes for both stands.

C) The last link of the investigations consists in ascertaining whether the difference in level between the two parallel regression planes may be supposed to have arisen by chance. We write the equations of the two planes

$$(14) \qquad \begin{cases} Z_1 = \bar{z}_1 + b\,(x - \bar{x}_1) + c\,(y - \bar{y}_1) \\ Z_2 = \bar{z}_2 + b\,(x - \bar{x}_2) + c\,(y - \bar{y}_2). \end{cases}$$

From these we find the difference in level

$$(15) \qquad d = Z_1 - Z_2 = \bar{z}_1 - \bar{z}_2 - b\,(\bar{x}_1 - \bar{x}_2) - c\,(\bar{y}_1 - \bar{y}_2).$$

This has the mean 0, and its variance (square of standard deviation) can be shown to be

$$(16) \qquad \operatorname{var}\{d\} = \sigma^2 \left(\frac{1}{n_1} + \frac{1}{n_2} + \frac{\varphi}{\varDelta}\right),$$

where

$$\varphi = (\bar{x}_1 - \bar{x}_2)^2 \sum_r SK_y - 2\,(\bar{x}_1 - \bar{x}_2)(\bar{y}_1 - \bar{y}_2) \sum_r SP_{xy} + (\bar{y}_1 - \bar{y}_2)^2 \sum_r SK_x$$

and

$$\varDelta = \sum_r SK_x \sum_r SK_y - (\sum_r SP_{xy})^2.$$

If we put

$$(19) \qquad \frac{1}{n_1} + \frac{1}{n_2} + \frac{\varphi}{\varDelta} = k,$$

$\dfrac{d}{s\sqrt{k}}$ will, d and s being independent, be t-distributed[1]) with

[1]) See Hald, p. 285 ff. and table IV.

$f_1 + f_2 = n_1 + n_2 - 6$ degrees of freedom, and by this means the hypothesis about coinciding regression planes can be tested.

In the example, (15) gives by calculation $d = 0.060$. Further we find by (17) and (18)

$$\varphi = 0.0009598, \qquad \Delta = 0.00978909,$$

after which (19) gives $k = 0.17993$, so that

$$t = \frac{d}{s\sqrt{k}} = 4.43.$$

From a t-table, it will be seen that with $n_1 + n_2 - 6 = 50$ degrees of freedom, $t_{0.975} = 2.01$; this corresponds to a two-sided symmetrical 95 % limitation. As the t-value found far exceeds 2.01, the hypothesis of a common level must absolutely be rejected.

§ 5. COMPARISON OF AN ARBITRARY NUMBER OF STANDS

In order to gain a general view of the material, we might imagine a comparison between all the stands by twos, made in the same way as described in § 4 for I 15 and I 7. This is a very slow method, and not the best suited, either. A general view is obtained by considering the whole material — or, at any rate, suitable larger parts of it — together, as described below.

To this must be added another point. Above we have examined the dependence of the total volume on the height h and the diameter d_0; but, as already mentioned in the introduction, we may also imagine the total volume to be dependent on other quantities, and it is, of course, important to include precisely those quantities which may be supposed to influence materially the total volume. As such quantities we have — in addition to h and d_0 — chosen the trunk quotient $\frac{d_{0.4}}{d_0}$, in which the Forest Experiment Station took a special interest, further the trunk form factor f and the branch volume G. We have carried out a regression for log T as a function of log h, log d_0, log $\frac{d_{0.4}}{d_0}$ [1]), log f, and log G, and as a function of only some of these. For each such set of variables we have computed the standard deviation. When the elimination

[1]) Replacement of log $\frac{d_{0.4}}{d_0}$ by log $d_{0.4}$ would have led to the same result (cf. § 2, the end).

of a variable has resulted in a substantial increase of the standard deviation, we have taken it to mean that this variable was of value in the determination of the total volume. The calculations show that actually only h and d_0 were of importance in that respect.

In the following presentation of the theory we will consider the case mentioned in § 4, where $z = \log T$ is regarded as a function of the two variables $x = \log h$ and $y = \log d_0$. On the whole it will be immediately obvious how the theory and formulas are altered by passing from two to more — or fewer — variables, and moreover for guidance the discription will be supplemented with brief comments on this point.

Let there be a total of k stands, containing $n_1, n_2, \ldots, n_k$ trees respectively. For each of these stands we form, as in § 4, the means $\bar{x}, \bar{y}, \bar{z}$, the sum of squares SK_x, SK_y, SK_z, and the product sums SP_{xy}, SP_{xz}, SP_{yz}. (If z is a function of m variables, we get $m + 1$ means, $m + 1$ square sums, and $\frac{1}{2} m (m + 1)$ product sums).

For each stand we then form[1]) the equations § 4, (4), and b and c are found; for the k stands they are denoted $(b_1\ c_1)$, (b_2, c_2), $\ldots$, (b_k, c_k).

Then, by means of § 4, (5), we form the quantities $q_1^2, q_2^2, \ldots, q_k^2$ with the degrees of freedom $f_1 = n_1 - 3$, $f_2 = n_2 - 3$, $\ldots$, $f_k = n_k - 3$, respectively, and

$$q^2 = q_1^2 + q_2^2 + \cdots + q_k^2 \tag{1}$$

with $f = \Sigma f_r$ degrees of freedom. (With m variables, § 4, (4) is replaced by m equations with m unknown, the number of the substraction terms in § 4, (5) is altered from 2 to m, and the degree of freedom for q^2_r is $n_r - m - 1$).

From § 4, (6) we form the quantities $s_1^2, s_2^2, \ldots, s_k^2$.

We now carry out the investigation according to the same classification A—C as is used in § 4 by comparison of the two stands; but within these subdivisions partially new and extended methods will be employed.

A) An investigation as to whether the differences between $s_1^2, s_2^2, \ldots, s_k^2$ may be of random origin. We form

$$s^2 = \frac{q^2}{f}, \tag{2}$$

[1]) Cf. the footnote on p. [14].

a kind of mean value of the k deviation squares found, and if we use Bartlett's test[1]), according to which

$$(3) \qquad \chi^2 = -\sum_{r=1}^{k} f_r \ln \frac{s_r^2}{s^2} \qquad ^{2)}$$

approximatively is χ^2-distributed with k—1 degrees of freedom.

With x and y denoting, as before log h and log d_0, the calculation, applied in the test, for the 17 stands in group I will appear from the following table 2, which is prepared on the basis of the calculations made for table 1 and the 16 tables analogous with that table.

Table 2.
To use by testing if the 17 stands of group I have a common standard deviation.

stand	number of trees	b_r	c_r	q_r^2	f_r	s_r^2	log s_r^2
I 1	31	0.7782	2.1844	0.022972	28	0.000820	0.9138—4
I 2	38	0.1725	1.9917	23172	35	662	0.8208
I 3	27	0.8843	2.2098	20252	24	844	0.9264
I 4	18	0.9526	2.0053	11384	15	759	0.8803
I 5	23	1.0739	1.9263	14488	20	724	0.8597
I 6	35	0.8434	2.1412	36985	32	1156	1.0629
I 7	28	0.3502	2.2624	26212	25	1048	1.0203
I 8	10	—0.1845	2.3383	02309	7	330	0.5185
I 9	12	0.5647	2.1999	13195	9	1466	1.1661
I 10	23	0.8424	2.0213	29497	20	1475	1.1688
I 11	25	0.8464	1.9877	25560	22	1162	1.0652
I 12	38	1.0067	2.0196	39604	35	1132	1.0538
I 13	30	0.8667	2.0405	45970	27	1703	1.2312
I 14	27	0.7937	1.8797	25754	24	1073	1.0306
I 15	33	0.4651	2.1649	27390	30	913	0.9604
I 16	14	0.8300	2.0779	10829	11	984	0.9930
I 17	31	0.8206	2.0231	27402	28	979	0.9908—4
	443 Σn_r			0.402975 q^2	392 Σf_r	0.001028 s^2	1.0120—4 log s^2

[1]) Hald, p. 246.
[2]) ln denotes natural logarithm.

The s^2 corresponding to (2) becomes

$$s^2 = \frac{0.402975}{392} = 0.001028.$$

We then get

$$\chi^2 = (\Sigma f_r) \ln s^2 - \Sigma (f_r \ln s_r^2) = \frac{1}{M} \left[(\Sigma f_r) \log s^2 - \Sigma (f_r \log s_r^2)\right],$$

where $M = \log e = 0.43429$. Accordingly

$$\chi^2 = \frac{1}{0.43429} (396.7040 - 389.7388) = \frac{6,9652}{0,43429} = 16.0.$$

From a χ^2-table[1]) it will be seen that with 17—1 = 16 degrees of freedom $\chi^2_{0.95} = 26.3$, corresponding to a one-sided 95 per cent limitation. The χ^2 found, which is considerably less than this value, is thus perfectly consistent with the hypothesis of a common standard deviation for all the 17 regressions, and, accordingly, this is assumed to be satisfied in the following. Corresponding calculations for Bregentved and Sorø give analogous results.

B) An investigation as to whether the regression planes may be regarded as parallel, that is, whether the differences between the pairs of figures (b_1, c_1), (b_2, c_2), . . , (b_k, c_k) may be of random character. The procedure is almost as in § 4. The formulas § 4, (9)—(12) may be used again, only each Σ-symbol should be interpreted as a summation of k terms, r running from 1 to k. The only real difference will be that q'^2, which previously had two degrees of freedom, now gets 2 (k—1) degrees of freedom, so that § 4, (13) must be replaced by

$$s'^2 = \frac{q'^2}{2\,(k-1)}. \tag{4}$$

The hypothesis of the parallelism of the planes is then tested by a comparison between the two independent estimates s'^2 and s^2 for the variance.

For the stands in group I considered above k = 17. From table 1 and the tables analogous with it we get

$$\Sigma\, SK_x = 0.344608, \quad \Sigma\, SK_y = 2.203872, \quad \Sigma\, SK_z = 11.719617$$
$$\Sigma\, SP_{xy} = 0.485676, \quad \Sigma\, SP_{xz} = 1.289269, \quad \Sigma\, SP_{yz} = 4.951325.$$

[1]) Cf. Hald, table V.

Hence the equations § 4, (9) become

$$0.344608\ b + 0.485676\ c = 1.289269$$
$$0.485676\ b + 2.203872\ c = 4.951325,$$

which have the solutions

$$(5) \qquad b = 0.833942 \qquad c = 2.062868.$$

By means of § 4, (11) and (12) we then find

$$\bar{q}^2 = 0.430512, \qquad q'^2 = 0.027537,$$

and (4) gives

$$s'^2 = \frac{0.27537}{32} = 0.000861.$$

As $s'^2 < s^2$, nothing prevents us from maintaining the hypothesis. As common values for the inclinations of the 17 regression planes we then use the values of b and c found in (5), and as an improved estimate of the variance we take

$$s^2 = \frac{q^2 + q'^2}{(n-3\,k) + 2\,(k-1)} = \frac{\bar{q}^2}{n - k - 2} = \frac{0.430512}{424} = 0.001015.$$

We will then compute the standard errors m_b and m_c of the b and c found. Employing the designations from § 4, (9), we get[1])

$$(6) \qquad m_b = \sqrt{\frac{\Sigma\, SK_y}{\Delta}} \cdot s, \quad m_c = \sqrt{\frac{\Sigma\, SK_x}{\Delta}} \cdot s,$$

where

$$(7) \qquad \Delta = \Sigma\, SK_x \cdot \Sigma\, SK_y - (\Sigma\, SP_{xy})^2.$$

Applied to the present example this gives $\Delta = 0.523591$ and then

$$(8) \qquad m_b = 0.0654, \qquad m_c = 0.0258.$$

For Bregentved and Sorø, also, it appears that the hypothesis of parallel regression planes can be maintained.

C) An investigation as to whether the parallel regression planes may be assumed to have a common level. As we have already ascertained that the levels for I 15 and I 7 are different, the object of the present investigation is to ascertain, whether stands normally have a common level, and the result previously

[1]) Cf. Hald, p. 503.

found was something exceptional for the two stands in question.

Here we consider the k "mean points" $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$, $(\bar{x}_2, \bar{y}_2, \bar{z}_2)$, .., $(\bar{x}_k, \bar{y}_k, \bar{z}_k)$ and try whether, with sufficient approximation, they lie in a plane which is parallel to the others. We then need symbols of the form

$$\bar{\bar{x}} = \frac{\Sigma\, n_r \bar{x}_r}{\Sigma\, n_r}, \quad SK_{\bar{x}} = \Sigma\, n_r (\bar{x}_r - \bar{\bar{x}})^2, \quad SP_{\bar{x}\bar{y}} = \Sigma\, n_r (\bar{x}_r - \bar{\bar{x}})(\bar{y}_r - \bar{\bar{y}}).$$

$\bar{\bar{x}}$ is the weighted mean for $\bar{x}_1, \bar{x}_2, .., \bar{x}_k$, the weights being the numbers n_r of trees in the individual stands; $\bar{\bar{x}}$ is then also the total mean of all the x's. $SK_{\bar{x}}$ and $SP_{\bar{x}\bar{y}}$ are formed on the basis of the individual means $\bar{x}_r$ and $\bar{y}_r$, similarly to sums of squares and sums of products in § 4, though with the figures n_r added as coefficients (weights).

The equations corresponding to § 4, (4) here become

$$(9) \qquad \begin{cases} b'\, SK_{\bar{x}} + c'\, SP_{\bar{x}\bar{y}} = SP_{\bar{x}\bar{z}} \\ b'\, SP_{\bar{x}\bar{y}} + c'\, SK_{\bar{y}} = SP_{\bar{y}\bar{z}}. \end{cases}$$

From these we find, by analogy with § 4, (5),

$$(10) \qquad q''^2 = SK_{\bar{z}} - b'\, SP_{\bar{x}\bar{z}} - c'\, SP_{\bar{y}\bar{z}}$$

with k—3 degrees of freedom, and from this we get the value s'' , independent of the previous ones, for the standard deviation determined by

$$(11) \qquad s''^2 = \frac{q''^2}{k-3}.$$

s'' is then to be compared with s.

For carrying out the calculations it is necessary to tabulate all the means $\bar{x}_r$, $\bar{y}_r$, $\bar{z}_r$, from which (using the weights n_r) we derive the sums of squares and products included in (9) and (10). For group I such a table gives

$$SK_{\bar{x}} = 0.7882, \quad SK_{\bar{y}} = 4.0981, \quad SK_{\bar{z}} = 24.6253,$$
$$SP_{\bar{x}\bar{y}} = 1.4018, \quad SP_{\bar{x}\bar{z}} = 3.6970, \quad SP_{\bar{y}\bar{z}} = 9.9749.$$

The equations (9) then become

$$0.7882\ b' + 1.4018\ c' = 3.6970$$
$$1.4018\ b' + 4.0981\ c' = 9.9749,$$

whence

$$b' = 0.92315 \qquad c' = 2.11825.$$

From (10) we then get $q''^2 = 0.0831$, accordingly

$$s''^2 = \frac{0.0831}{14} = 0.005936.$$

Hence

$$v^2 = \frac{s''^2}{s^2} = \frac{0.005936}{0.001028} = 5.77.$$

The use of a v^2-table shows that with 14 and 392 degrees of freedom in the numerator and denominator, respectively, $v^2_{0.95} = 1.72$, corresponding to a one-sided 95 per cent limitation. As the v^2 found is much larger than this, the hypothesis of a common level must be absolutely rejected.

For the Bregentved and Sorø districts, also, the calculations show that the hypothesis of o common level cannot be maintained. However, the differences which bring about the rejection, notably as regards Bregentved, are not nearly as large as for group I.

§ 6. COMPILATION OF THE WHOLE MATERIAL

In the preceding section we have mentioned separately the treatment of the three main parts of the material, group I, Bregentved, and Sorø. Within each of these main parts certain characteristic facts have appeared, which, after elimination of random variations, may be expressed as follows:

A) The standard deviations from the regression planes are equal.

B) The regression planes are parallel.

C) The regression planes do not coincide, the levels being different.

We will now examine whether conditions A) and B) may be extended so as to be valid for the whole material. In the following table we then compare the results found in § 5 for group I with the corresponding results for Bregentved and Sorø:

Table 3.

	b	c	s^2	s	m_b	m_c
Group I	0.8339	2.0629	0.001015	0.0319	0.0654	0.0258
Bregentved	0.4153	2.1439	0.000625	0.0250	0.1270	0.0545
Sorø	0.8519	1.9853	0.000679	0.0261	0.1474	0.0592

It will be seen that the standard deviation s for the sample plots differs rather considerably from the other two, and Bartlett's test, as applied in § 5, shows that this difference cannot be explained as accidental. The s-values for Bregentved and Sorø, however, are almost equal, and, as mentioned in § 4 under A), a v^2-test shows that the small difference may quite well be explained as due to chance. The assumption of a common standard deviation from the regression planes may thus be maintained by comparison between Bregentved and Sorø, but not if the comparison is extended to group I, also.

This result is in natural agreement with the fact that in the Bregentved and Sorø districts the selection of trees must be assumed to have taken place according to almost the same principles, whereas group I holds a more exceptional position. For, while the observations from the former two districts were collected by the district administration and were the result of the selection of sample trees, the material of group I was gathered by the Forest Experimental Station and is derived from clean cutting as well as from selection within the stands, but in the latter case the whole stand spectrum is represented (cf. Medd. 149, p. 5 ff.).

We will now proceed to examine how it is with the parallelism of the regression planes, that is to say, we will endeavour to find out whether the differences between the b-values as well as between the c-values in table 3 may be of random character. A first glance shows that the c-values are fairly equal, whereas the b-values for Bregentved differ a good deal from the others.

We will then, as shown on p. 16, first work the two s^2-values for Bregentved and Sorø into a common value; this becomes

$$s^2 = 0.000648.$$

The ratio between s^2 for group I and that found above is

$$\frac{0.001015}{0.000648} = 1.57.$$

We therefore reduce the sums of squares and products of group I by dividing them by 1.57. The values for b and c are not altered by this; but the variance s^2 is reduced in the said proportion and thus becomes equal to the common value for Bregentved and Sorø. This is the same as using the reciprocal values of the variances as weights on the sums of squares and products for

Bregentved-Sorø and group I. The three main parts can now be compared, and a v^2-test, similar to that mentioned in § 5 under B), shows that it will be justifiable to regard the three b-values, as well as the three c-values, as equal. The total result then becomes

Table 4.

	b	c	s^2	s	m_b	m_c
Group I Bregentved-Sorø	0.7636	2.0640	0.001015	0.0319	0.0540	0.0217
			0.000648	0.0255		

§ 7. THE CALCULATION METHOD EMPLOYED

In the preceding sections the calculations have been carried out for the case that log T is regarded as a function of log h and log d_0. It was mentioned, however, that it was useful to include several independent variables in the investigations in order to find such, if any, which in addition to h and d_0 were of essential importance for the determination of the total volume. In this way the work of calculation is considerably increased, and it is therefore of importance to employ standard methods which reduce the work as much as possible. We have decided to use Cholesky's method[1]).

We will describe this method applied to the case previously considered, where log T was determined as a function of log h and log d_0; but in order to facilitate the possibility of transition to more variables, the independent variables log h and log d_0 will be denoted x_1 and x_2, while the dependent variable log T will be denoted y. As regards the sums of squares and products, also, we introduce a minor alteration, thus instead of e. g. SK_{x_1}, $SP_{x_1x_2}$, and SP_{x_1y} we write SK_1, SP_{12}, and SP_{1y}. Finally, we write a_1 and a_2 instead of b and c. The equations § 4, (4) then become

$$(1) \qquad \begin{cases} a_1 SK_1 + a_2 SP_{12} = SP_{1y} \\ a_1 SP_{12} + a_2 SK_2 = SP_{2y}, \end{cases}$$

and § 4, (5) becomes

$$(2) \qquad q^2 = SK_y - a_1 SP_{1y} - a_2 SP_{2y}.$$

[1]) Bull. géod. 2, 1924. Cf. also Henry Jensen, An Attempt at a Systematic Classification of some Methods for the Solution of Normal Equations. Geod. Inst. Medd. 18, 1942.

According to the method previously mentioned, (1) is first solved as to the a's, after which these, by insertion in (2), give the value for q^2. In the method actually employed (and described below) this sequence is reversed.

The calculations are most easily carried out in association with a matrix equation

$$(3)\quad \begin{array}{c} \left\{\begin{matrix} b_{11} & 0 & 0 \\ b_{12} & b_{22} & 0 \\ d_1 & d_2 & d_3 \end{matrix}\right\} \\ \hline k'_1 \quad k'_2 \quad k'_3 \end{array} \left\{\begin{matrix} b_{11} & b_{12} & d_1 \\ 0 & b_{22} & d_2 \\ 0 & 0 & d_3 \end{matrix}\right\} = \left\{\begin{matrix} SK_1 & SP_{12} & SP_{1y} \\ SP_{12} & SK_2 & SP_{2y} \\ SP_{1y} & SP_{2y} & SK_y \end{matrix}\right\} \begin{matrix} k_1 \\ k_2 \\ k_3, \end{matrix}$$

where the b's and d's on the left hand side are auxiliary quantities, which are to be found, while the square sums and the product sums must be imagined to be calculated from the experimental material, as previously mentioned. The quantities k_1, k_2, k_3 denote the sums of the rows of the matrix to the right; they are easily computed. Analogously, k'_1, k'_2, k'_3 denote the sums of the columns of the first matrix to the left; they are used for checking the calculations.

Equation (3) should be interpreted so that the product sum of the elements in the pth row of Matrix 1 and the elements in the qth column of Matrix 2 shall be equal to the element of Matrix 3 situated in the pth row and the qth column ("row-column multiplication"). Thus we get

$$(4)\quad \begin{array}{llll} \text{Row 1 multiplied by column 1:} & & b^2_{11} & = SK_1 \\ \text{— 2 — - — 1:} & & b_{12}\, b_{11} & = SP_{12} \\ \text{— 3 — - — 1:} & & d_1\, b_{11} & = SP_{1y} \\ \text{— 4 — - — 1:} & & k'_1\, b_{11} & = k_1. \end{array}$$

From these equations we determine b_{11}, b_{12}, d_1, and k_1, and we check whether

$$(5)\qquad b_{11} + b_{12} + d_1 = k'_1.$$

Then we form

$$(6)\quad \begin{array}{llll} \text{Row 2 multiplied by column 2:} & & b_{12}^2 + b_{22}^2 & = SK_2 \\ \text{— 3 — - — 2:} & & d_1\, b_{12} + d_2\, b_{22} & = SP_{2y} \\ \text{— 4 — - — 2:} & & k'_1 b_{12} + k'_2 b_{22} & = k_2 \end{array}$$

Hence we find b_{22}, d_2, and k'_2, and we check whether

$$b_{22} + d_2 = k'_2. \tag{7}$$

Finally we form

$$\begin{array}{lll} \text{Row 3 multiplied by column 3:} & d_1^2 + d_2^2 + d_3^2 & = SK_y \\ \text{— 4 — - — 3:} & k'_1 d_1 + k'_2 d_2 + k'_3 d_3 & = k_3. \end{array} \tag{8}$$

Hence d_3 and k'_3, and it is checked whether

$$d_3 = k'_3. \tag{9}$$

Now the following rule will apply: The quantities

$$SK_y = d_1^2 + d_2^2 + d_3^2, \qquad SK_y - d_1^2 = d_2^2 + d_3^2,$$
$$SK_y - d_1^2 - d_2^2 = d_3^2$$

constitute the square sum q^2 for, respectively, the y's alone, y as a function of x_1, and y as a function of x_1 and x_2. If we divide by the corresponding number of degrees of freedom ($n - 1$, $n - 2$, and $n - 3$, the number of trees being n), we get the corresponding variances s^2, the square roots of which are the standard deviations s.

If we then wish to determine the regression coefficients a_1 and a_2 in (1), this may most easily be done by means of the matrix equation

$$\begin{Bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{Bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}, \tag{10}$$

which is equivalent to the two equations

$$\begin{aligned} b_{11}\, a_1 + b_{12}\, a_2 &= d_1 \\ b_{22}\, a_2 &= d_2. \end{aligned} \tag{11}$$

To illustrate the method we again make the calculations for I 15. Here the equation (3) according to previously found sums of squares and products becomes

$$\begin{array}{c} \begin{Bmatrix} b_{11} & 0 & 0 \\ b_{12} & b_{22} & 0 \\ d_1 & d_2 & d_3 \end{Bmatrix} \\ \hline k'_1 \; k'_2 \; k'_3 \end{array} \begin{Bmatrix} b_{11} & b_{12} & d_1 \\ 0 & b_{22} & d_2 \\ 0 & 0 & d_3 \end{Bmatrix} = \begin{Bmatrix} 0.028297 & 0.039571 & 0.106308 \\ 0.039571 & 0.083279 & 0.204695 \\ 0.106308 & 0.204695 & 0.523129 \end{Bmatrix} \begin{Bmatrix} 0.174176 \\ 0.327545 \\ 0.834132 \end{Bmatrix} \tag{12}$$

The first set of equations (4) gives

$b_{11} = 0.168217$, $b_{12} = 0.235238$, $d_1 = 0.631969$, $k'_1 = 1.035424$,

and it will be seen that the test equation (5) is satisfied. The next set of equations (6) then gives

$$b_{22} = 0.167159, \quad d_2 = 0.335201, \quad k'_2 = 0.502360,$$

and it is seen that (7) is satisfied. Finally, the equations (8) give

$$d_3 = 0.106698, \quad k'_3 = 0.106699,$$

so that the last checking is in order, also. We then have

q^2	0.523129	0.123744	0.011384
f	17	16	15
s^2	0.030772	0.007734	0.000759
s	0.1754	0.0879	0.0275.

The standard deviation for the y's alone is accordingly 0.1754, by regression with regard to x_1 it is reduced to 0.0879, and if further x_2 is added, it drops right down to 0.0275 (cf. § 4, p. 15).

The regression coefficients a_1 and a_2 are now determined from (11), which becomes

$$\begin{aligned} 0.168217\ a_1 + 0.235238\ a_2 &= 0.631969 \\ 0.167159\ a_2 &= 0.335201. \end{aligned} \tag{13}$$

Hence

$$a_1 = 0.952639, \quad a_2 = 2.005282$$

in accordance with the previous calculations.

The matrix equation (3) may be written in a much more condensed form. Thus we may omit the corner of the first matrix on the left hand side which consists of zeros only, as well as the whole second matrix. Further, we may omit the corner below, left, of the matrix on the right hand side, so that the product sums are only written once. We then get the following diagram:

b_1	SK_1	SP_{12}	SP_{1y}	k_1
b_{12}	b_{22}	SK_2	SP_{2y}	k_2
d_1	d_2	d_3	SK_y	k_3
k'_1	k'_2	k'_3		

In the above example of the calculation it appears as follows:

	1	2	y	sum
0.168217	0.028297	0.039571	0.106308	0.174176
0.235238	0.167159	0.083279	0.204695	0.327545
0.631969	0.335201	0.106696	0.523129	0.834132
1.035424	0.502360	0.106696		

As an example of the calculation in case of more independent variables we give the diagram and the results obtained from it for log T as a function of log h, log d_0, log $\frac{d_{0.4}}{d_0}$, and log G, likewise for I 15:

	1	2	3	4	y	sum
0.168217	0.028297	0.039571	−0.003377	0.111155	0.106308	0.281954
0.235238	0.167159	0.083279	−0.013445	0.244843	0.204695	0.558943
−0.020075	−0.052181	0.111185	0.015488	−0.050222	−0.026248	−0.077804
0.660783	0.534830	−0.081385	0.385016	0.877538	0.628939	1.812253
0.631969	0.335201	0.035346	0.090764	0.043555	0.523129	1.436823
1.676133	0.985007	0.065144	0.475781	0.043555		

Hence:

q^2	0.523129	0.123744	0.011384	0.010135	0.001897
f	17	16	15	14	13
s^2	0.030772	0.007734	0.000759	0.000724	0.000146
s	0.1754	0.0879	0.0275	0.0269	0.0121

The first diagram, which only comprises x_1 and x_2, enters as part of the second, and the former results are likewise included in the latter. Equations (13), also, may be deduced from the last diagram given. As before, it will be seen that the introduction of the variables x_1 and x_2 considerably reduces the standard deviation. The inclusion of x_3 leaves it almost unaltered; if, finally, x_4 is added, it decreases again, but to a small extent, only.

In table 5 a series of standard deviations are given for all the stands in group I. The first column from the left corresponds to the total variation of y = log T, the next one the regression with regard to x_1 = log h, and the following ones to a successive ad-

dition of $x_2 = \log d_0$, $x_3 = \log \frac{d_{0.4}}{d_0}$, and $x_4 = \log G$ as variables. Thus the last column gives the standard deviation of a regression in which y is a function of all the four variables mentioned.

Table 5.
Standard deviations for the 17 stands of group I by regression with regard to several variables.

stand		x_1	x_1 x_2	x_1 x_2 x_3	x_1 x_2 x_3 x_4
I 1	0.2267	0.1298	0.0286	0.0272	0.0130
I 2	1067	953	257	220	149
I 3	1501	1094	291	204	157
I 4	1754	879	275	269	121
I 5	1435	1088	269	260	189
I 6	1459	948	340	306	224
I 7	2011	1258	324	247	151
I 8	1703	1198	182	91	91
I 9	1666	1472	383	406	113
I 10	2184	1811	384	318	201
I 11	2131	1644	341	249	174
I 12	1324	1226	336	171	137
I 13	1604	1322	413	259	147
I 14	1160	1073	328	315	242
I 15	1112	1013	302	245	163
I 16	2386	1418	314	214	120
I 17	1624	1490	313	302	183
total ($\bar{s}$)	0.1659	0.1274	0.0319	0.0272	0.0176

Thus, what was said about I 15 as to the variability of the standard deviation with the addition of the various variables, applies to all the stands. Corresponding tables prepared for the Bregentved and Sorø districts show analogous conditions.

§ 8. VOLUME DETERMINATION FOR A STAND

We shall now mention how, on the basis of the above, we may arrive at a volume determination for a stand. Corresponding to each of the $k = 17 + 11 + 8 = 36$ stands considered above we have found a regression plane of the form

$$(1) \qquad Z = \bar{z} + b\,(x - \bar{x}) + c\,(y - \bar{y}),$$

where $\bar{x}$, $\bar{y}$, and $\bar{z}$ denote the means of the x-, y-, and z-values, respectively, for the particular stand. The k planes may be regarded as parallel, and their common direction may be fixed at the values mentioned at the end of § 6, viz.

$$(2) \qquad b = 0.7636, \quad c = 2.0640.$$

However, the planes lie at different levels. These levels can be compared if in all the planes (1) we insert one and the same point (x, y) and compute the corresponding Z-values, Z_1, Z_2, . . ., Z_k. As the common point we have chosen below $(\bar{\bar{x}}, \bar{\bar{y}})$, where $\bar{\bar{x}}$ and $\bar{\bar{y}}$ denote the total means of all the x's, respectively all the y's, in all the stands. The corresponding Z-values (levels) are listed in table 6.

As a mean of the 17 levels in group I we have $\bar{Z} = \frac{3.5992}{17} = 0.2117$. The corresponding plane of type (1) is

$$(3) \qquad Z = \bar{Z} + b\,(x - \bar{\bar{x}}) + c\,(y - \bar{\bar{y}}).$$

Here x, y, and Z stand for the logarithms of a tree's diameter d_o, its height h, and its total volums T (determined by d_o and h). This dependence is tabulated in the volume table given on p. [39] —[44], where when entering with d_o and h we may find the corresponding T. The values of d_o and h are indicated with an interval of ½ cm and ½ m, respectively.

We shall now see how this volume table can be employed to find the total volume for a particular stand. Let us suppose that the diameter and height have been measured for all the trees in the stand. If the regression plane (1) for the stand corresponds precisely to the selected mean level $\bar{Z}$, the volume table may be directly used, and from the measured diameters and heights give the corresponding volumes. If, however, as must normally be assumed, the level of the stand differs from $\bar{Z}$, a correction must be made corresponding to a vertical displacement of the plane (3). In this way all the Z-values receive the same increase (or reduction), and since Z = log T, the corresponding T-values will all be increased by the same percentages, p. The figure p can be determined in the following way:

We imagine that a small number, n, of trees are felled. The

Table 6.
Z-values (levels) for all 36 stands.

Group I	Z_i
I 1	0.2260
I 2	0.2370
I 3	0.2268
I 4	0.2383
I 5	0.1904
I 6	0.2265
I 7	0.2191
I 8	0.2165
I 9	0.2149
I 10	0.1793
I 11	0.1869
I 12	0.1877
I 13	0.2222
I 14	0.2005
I 15	0.2166
I 16	0.2015
I 17	0.2090
sum	3.5992
mean	0.2117
total mean $\frac{7.2367}{36} = 0.2010$	

Group II (Bregentved)	Z_i
II 1	0.2221
II 2	0.2162
II 3	0.1995
II 4	0.1977
II 5	0.1935
II 6	0.2069
II 7	0.2178
II 8	0.1940
II 9	0.2017
II 10	0.1816
II 11	0.2016
sum	2.2326
mean	0.2030

Group III (Sorø)	Z_i
III 1	0.1839
III 2	0.2118
III 3	0.1636
III 4	0.1487
III 5	0.1809
III 6	0.1546
III 7	0.1872
III 8	0.1742
sum	1.4049
mean	0.1756

actual total volumes of these are measured and denoted $t_1, t_2, \ldots, t_n$. Let the volume table for the total volumes of the same trees give the values $T_1, T_2, \ldots, T_n$. If the first-mentioned volumes exceed the calculated ones by $p_1, p_2, \ldots, p_n$ per cent, respectively, p becomes the average

$$p = \frac{1}{n}(p_1 + p_2 + \ldots + p_n). \tag{4}$$

If then, by consulting the volume table, the total volume is deter-

mined for all the standing trees one at a time, we merely have to add all the total volumes thus found and increase the result by p per cent. In this way the total volume of the whole stand is found, corrected with regard to the level of the stand.

For the accuracy of this volume T the following formula applies:

$$m_T = 2.30 \cdot T \sqrt{\frac{s^2}{n} + \frac{s^2}{N}}, \tag{5}$$

where s indicates the previously found standard deviation from the regression plane, which may approximately be put at 0.03, while n, as mentioned above, is the number of trees felled whose volume has been directly determined, and N is the number of standing trees in the stand. The first term under the radical sign is an expression of the inaccuracy of the determination of level and may be reduced by increasing n; the second term covers the random biological variation between the trees, and if N is large, we may disregard this element, so that (5) is simplified to

$$m_T = 2.30 \cdot T \cdot \frac{s}{\sqrt{n}}. \tag{6}$$

If, for example, we put $n = 9$, we get from this

$$m_T = 2.30 \cdot T \cdot \frac{0.03}{3},$$

that is, the mean error of T is 2.3 per cent. Similarly, for $n = 4$ we get a mean error of 3.5 per cent.

It is an assumption for the validity of (5) and (6) that the means $\bar{x}$ and $\bar{y}$ for the n felled trees shall approximately be equal to the corresponding quantities for the whole forest; if not, the inaccuracy will be increased.

Even without the said direct volume determination for some few trees, i. e. by measurement of diameters and heights alone, we may, on the basis outlined above, form an estimate of the total volume of a particular stand, but the accuracy will then be less.

Since in such case we cannot determine any correction per cent p, we use the volume table as it is, that is, for want of better knowledge we assume that the level of the stand is the mean level. An estimation of the accuracy is then obtained by considering the $k = 36$ levels $Z_1, Z_2, \ldots, Z_{36}$ with the average $Z = 0.2010$ and calculating their mean error r from the formula

$$(7) \qquad r^2 = \frac{\Sigma (Z_i - \bar{\bar{Z}})^2}{k-1} = \frac{\Sigma Z_i^2 - \frac{1}{k}(\Sigma Z_i)^2}{k-1} .$$

We then replace $\frac{s^2}{n}$ by r^2 as the first term under the radical sign in (5) and get

$$(8) \qquad m_T = 2.30\ T \sqrt{r^2 + \frac{s^2}{N}},$$

which for large values of N is simplified to

$$(9) \qquad m_T = 2.30\ T\ r.$$

From table 6 we find $r^2 = 0.0004744$, accordingly $r = 0.022$, so that (9) becomes

$$m_T = 2.30\ T \cdot 0.022 = 0.051\ T,$$

that is to say, the standard error of T is 5 per cent.

Consequently, here is a limit to the accuracy, below which we cannot get by the usual employment of volume tables with entry by means of diameter and height. If, however, we undertake a direct determination of the volume for some few trees, the level of the stand will thereby be approximately established, and in the way described above, by using a correction per cent, more accurate results will be obtained, after examination of e. g. 9 trees, as mentioned above, with a standard error of 2.3 per cent.

For further application of the table see report 172.

As to the practical use of the tables the following remarks should be added regarding *interpolation in the table.* If the measured height and diameter are not stated directly in the table, it is necessary to interpolate. This may be done as shown in the example below:

Height 25.2 m, diameter 44.9 cm.

25.0 and 44.0 correspond to the wood volume		2.308 m^3
Increase in height:	$^2/_5$ (2.343 — 2.308) =	0.014 -
— - diam. :	$^9/_{10}$ (2.417 — 2.308) =	0.098 -
	Result	2.420 m^3

§ 9. FORKED TREES

In the calculation of the table, forked trees beginning at $d_{0.6}$ or farther down have not been included. The limit was put at $d_{0.6}$, an investigation having shown that forked trees from $d_{0.7}$ and upwards have no discernible deviation in level from unforked trees.

For the other forked trees, the level is increasing — the greater the deeper the fork is located. To obtain a rule from which to start, the correction factor was computed for the forked trees as well as for the other trees in a number of stands (see the subjoined table):

Stand	Forked trees	Other trees
Brahetrolleborg	1.035	1.026
Boller	1.110	1.051
Giesegaard	0.996	0.947
Esrum 281	1.014	1.014
Frederiksdal	1.038	0.928
Merløse 21	1.022	0.998
Petersgaard	1.038	1.015
Stensballe	1.096	0.997
Esrum 307	1.061	1.016
Sorø	1.046	0.986
Aastrup	1.007	0.951

From this it appears that a reasonable estimate for the correction factor of the forked trees may be obtained by adding 0.05 to the correction factor for the other trees. If the percentage of forked trees in the stand is known, i. e. the percentage of trees in the stand which are forked from $d_{0.6}$ or lower down, the correction factor can be calculated for the whole stand as the correction factor already found plus 0.05 times p/100, where p is the percentage of forked trees. This gives

Forked tree percent	Addition to the correction factor
5	0.0025
10	0.0050
15	0.0075
20	0.0100
25	0.0125

SAMMENDRAG.

Almindelig Massetabel for Bøg i Danmark.

I det foregående er der meddelt nogle resultater af en undersøgelse vedrørende bøgens vedmasse. Det benyttede materiale er stillet til rådighed af Statens forstlige Forsøgsvæsen, hvor afdelingsleder H. A. Henriksen under hele udarbejdelsen har ydet forfatterne værdifuld forstsagkyndig bistand. Materialet stammer fra Danmarks bøgebevoksninger og falder naturligt i tre grupper. Gruppe I består af målinger udført af Forsøgsvæsenet på 17 bevoksninger, fordelt over landets forskellige dele og bestående af fra 10 til 38 træer, medens gruppe II og III omfatter målinger fra en række bevoksninger, henholdsvis 11 og 8, fra Bregentved og Sorø skovdistrikter, bestående af fra 6 til 15 træer. Materialerne i gruppe II og III er tilvejebragt af distriktsadministrationerne.

Træerne i gruppe I er målt efter Forsøgsvæsenets sædvanlige metode til måling af prøvetræer. Volumen af stammedelen under 1,3 m er bestemt ved mål på midten af fire lige store sektioner, medens volumen af stammedelen over 1,3 m er bestemt ved endeflademål på 10 lige store sektioner. Grenemassen er bestemt ved klupning i 1 m-sektioner, og kvasmængden (diam. under 3 cm) ved vejning. Træerne i gruppe II og III er målt på en lidt anden måde. Den væsentlige forskel består i, at også stammedelen over 1,3 m er målt i 1 eller 2 m-sektioner.

Hovedvægten af undersøgelsen ligger i bestemmelse og undersøgelse af regressionsplaner mellem højde, diameter og vedmasse for hver af de undersøgte bevoksninger.

Ved overgang til logaritmisk mål for disse størrelser viser det sig, at regressionsfladerne, som i naturligt mål er krumme, ændres til planer (sml. fig. 1—8, s. [7]—[8]). Yderligere opnår man, at spredningen omkring regressionsplanerne bliver den samme for store og små træer, medens spredningen i naturligt mål er størst for de store træer.

Det er påvist, at der intet er i vejen for at betragte de enkelte iagttagelsers spredning omkring planerne som ens for de forskellige bevoksninger i gruppe I (s. [19]—[21]). Derimod er spredningen mindre for bevoksningerne i gruppe II og III. Denne forskel er sandsynligvis ikke reel, men kan formentlig henføres til materialernes forskellige oprindelse og deraf følgende forskellige beskaffenhed: Materialet i gruppe I, der er indsamlet af Forsøgsvæsenet, er dels fremkommet ved renafdrift, dels også ved udtagning i bevoksninger, men da således, at hele bevoksningsspektret er tilstræbt repræsenteret; det er da rimeligt, om det indeholder flere forskellige typer end materialerne i gruppe II og III, som er fremkommet ved prøvetræudtagninger. I almindelighed kan man (i logaritmisk mål) regne med en spredning s af de enkelte iagttagelser omkring regressionsplanen på 0,03; dette svarer for selve vedmassen til ca. 7 %.

Det er endvidere undersøgt, om man kan anse regressionsplanerne for de forskellige bevoksninger for at være parallelle. Dette spørgsmål besvares bekræftende (s. [21]—[22], [25]—[26]).

Endelig er det påvist, at man ikke kan anse de forskellige bevoksningers regressionsplaner for at være sammenfaldende (s. [22]—[24]). Man siger, at planerne har forskelligt niveau. Disse niveauer har en spredning, der omsat fra logaritmisk til naturligt mål beløber sig til ca. 5 % af vedmassen.

Ud fra hele det foreliggende materiale er bestemt en regressionsplan, hvis niveau er middelværdien af de 17 niveauer i gruppe I, og på grundlag af denne er fremstillet en massetavle (s. [39]), der tjener til beregning af en bevoksnings vedmasse. Tavlen benyttes på følgende måde: Først bestemmes niveauet ved prøvetræmålinger, idet man sammenholder prøvetræernes virkelige totalmasser t_1, t_2 ...., t_n med de masser T_1, T_2, ..., T_n, man finder i massetavlen ved indgang med højde og diameter. Hvis de førstnævnte masser overgår de beregnede med p_1, p_2, ..., p_n %, er middeltallet $p = \frac{1}{n}(p_1 + p_2 + \ldots + p_n)$ et udtryk for niveauet. Bestemmes herefter ved opslag i massetavlen totalmassen for alle stående træer eet ad gangen, har man blot at addere de således fundne totalmasser og forøge resultatet med p %. Herved er fundet bevoksningens samlede totalmasse, korrigeret under hensyn til dens niveau.

Er som anført antallet af prøvetræer n, medens det samlede antal af træer i bevoksningen er meget stort, bliver vedmassen bestemt ved en middelfejl m_T, der tilnærmelsesvis er $230 \frac{s}{\sqrt{n}}$ %. Med benyttelse af værdien $s = 0{,}03$ bliver altså $m_T = \frac{7}{\sqrt{n}}$ %. Måles f. eks. 9 prøvetræer ($n = 9$), fås $m_T = 2{,}3$ %. I dette tilfælde vil man altså kunne påregne en nøjagtighed på ca. 4,6 %, når der regnes med 95 %'s sikkerhedsinterval.

Også uden benyttelse af prøvetræer kan massetavlen anvendes. Regner man da med, at bevoksningen har middelniveau, indfører man en fejlkilde, der kan bevirke afvigelser på indtil ca. 10 %.

En uddybning heraf samt andre metoder til anvendelse af massetavlen omtales i beretning 172, s. [61] ff.

Table 7 (report 171), Tabelle I (Bericht 172).

Total volume according to the general volume surface for beech in Denmark.

Baummassen entsprechend der generellen Massenfläche für Buche in Dänemark.
Totalmasser gældende for den generelle masseflade for bøg i Danmark.

Height Höhe Højde m	Diameter b.h. - Durchmesser - Diameter cm										Height Höhe Højde m
	5	6	7	8	9	10	11	12	13	14	
	Total volume, cubic meter - Baummassen, fm - Totalmasse, m^3										
9	0,0119	0,0173	0,0238	0,0314	0,0401	0,0497	0,0606	0,0724	0,0855	0,0996	9
9,5	0,0124	0,0181	0,0248	0,0327	0,0417	0,0518	0,0631	0,0755	0,0891	0,104	9,5
10	0,0129	0,0188	0,0258	0,0340	0,0434	0,0539	0,0656	0,0785	0,0927	0,108	10
10,5	0,0134	0,0195	0,0268	0,0353	0,0450	0,0559	0,0681	0,0815	0,0961	0,112	10,5
11		0,0202	0,0278	0,0366	0,0466	0,0580	0,0706	0,0845	0,0996	0,116	11
11,5		0,0209	0,0287	0,0378	0,0482	0,0600	0,0730	0,0874	0,103	0,120	11,5
12		0,0216	0,0297	0,0391	0,0498	0,0619	0,0754	0,0902	0,106	0,124	12
12,5		0,0223	0,0306	0,0403	0,0514	0,0639	0,0778	0,0931	0,110	0,128	12,5
13		0,0230	0,0315	0,0415	0,0530	0,0658	0,0802	0,0959	0,113	0,132	13
13,5		0,0236	0,0324	0,0428	0,0545	0,0678	0,0825	0,0987	0,116	0,136	13,5
14		0,0243	0,0334	0,0440	0,0561	0,0697	0,0849	0,102	0,120	0,140	14
14,5		0,0250	0,0343	0,0451	0,0576	0,0716	0,0872	0,104	0,123	0,143	14,5
15		0,0256	0,0352	0,0463	0,0591	0,0734	0,0894	0,107	0,126	0,147	15
15,5		0,0262	0,0361	0,0475	0,0606	0,0753	0,0917	0,110	0,129	0,151	15,5
16		0,0269	0,0370	0,0487	0,0621	0,0771	0,0940	0,112	0,133	0,155	16
16,5		0,0275	0,0378	0,0498	0,0636	0,0790	0,0962	0,115	0,136	0,158	16,5
17			0,0387	0,0510	0,0650	0,0808	0,0984	0,118	0,139	0,162	17
17,5			0,0396	0,0521	0,0665	0,0826	0,101	0,120	0,142	0,166	17,5
18			0,0404	0,0532	0,0679	0,0844	0,103	0,123	0,145	0,169	18
18,5			0,0413	0,0544	0,0694	0,0862	0,105	0,126	0,148	0,173	18,5
19				0,0555	0,0708	0,0879	0,107	0,128	0,151	0,176	19
19,5				0,0566	0,0722	0,0897	0,109	0,131	0,154	0,180	19,5
20					0,0736	0,0914	0,111	0,133	0,157	0,183	20
20,5					0,0750	0,0932	0,114	0,136	0,160	0,187	20,5
21						0,0949	0,116	0,138	0,163	0,190	21
21,5						0,0966	0,118	0,141	0,166	0,194	21,5
22							0,120	0,143	0,169	0,197	22
22,5							0,122	0,146	0,172	0,201	22,5
23								0,148	0,175	0,204	23
23,5								0,151	0,178	0,207	23,5
24								0,153	0,181	0,211	24
24,5								0,156	0,184	0,214	24,5
25									0,186	0,217	25
25,5									0,189	0,221	25,5

Table 7 (continued).

Tabelle I (Fortsetzung).

Height Höhe Højde m	Diameter b.h. - Durchmesser - Diameter cm												Height Höhe Højde m
	15	16	17	18	19	20	21	22	23	24	25	26	
	Total volume, cubic meter - Baummassen, fm - Totalmasse, m^3												
9	0,115	0,131											
9,5	0,120	0,137	0,155										
10	0,125	0,142	0,161										
10,5	0,129	0,148	0,167										
11	0,134	0,153	0,173										
11,5	0,138	0,158	0,179	0,202	0,226	0,251	0,277	0,305	0,334	0,365			
12	0,143	0,163	0,185	0,208	0,233	0,259	0,286	0,315	0,346	0,377			
12,5	0,148	0,169	0,191	0,215	0,240	0,267	0,295	0,325	0,356	0,389			
13	0,152	0,174	0,197	0,221	0,248	0,275	0,304	0,335	0,367	0,401			
13,5	0,156	0,179	0,203	0,228	0,255	0,283	0,313	0,345	0,378	0,413	0,449	0,487	13,5
14	0,161	0,184	0,208	0,234	0,262	0,291	0,322	0,355	0,389	0,424	0,462	0,501	14
14,5	0,165	0,189	0,214	0,241	0,269	0,299	0,331	0,364	0,399	0,436	0,474	0,514	14,5
15	0,170	0,194	0,220	0,247	0,276	0,307	0,340	0,374	0,410	0,447	0,487	0,528	15
15,5	0,174	0,199	0,225	0,253	0,283	0,315	0,348	0,383	0,420	0,459	0,499	0,541	15,5
16	0,178	0,204	0,231	0,259	0,290	0,323	0,357	0,393	0,430	0,470	0,511	0,554	16
16,5	0,182	0,209	0,236	0,266	0,297	0,330	0,365	0,402	0,441	0,481	0,523	0,568	16,5
17	0,187	0,213	0,242	0,272	0,304	0,338	0,374	0,411	0,451	0,492	0,535	0,581	17
17,5	0,191	0,218	0,247	0,278	0,311	0,345	0,382	0,420	0,461	0,503	0,547	0,594	17,5
18	0,195	0,223	0,252	0,284	0,317	0,353	0,390	0,430	0,471	0,514	0,559	0,607	18
18,5	0,199	0,227	0,258	0,290	0,324	0,360	0,399	0,439	0,481	0,525	0,571	0,619	18,5
19	0,203	0,232	0,263	0,296	0,331	0,368	0,407	0,448	0,491	0,536	0,583	0,632	19
19,5	0,207	0,237	0,268	0,302	0,337	0,375	0,415	0,457	0,500	0,546	0,594	0,645	19,5
20	0,211	0,241	0,273	0,308	0,344	0,382	0,423	0,465	0,510	0,557	0,606	0,657	20
20,5	0,215	0,246	0,279	0,313	0,350	0,390	0,431	0,474	0,520	0,568	0,618	0,670	20,5
21	0,219	0,251	0,284	0,319	0,357	0,397	0,439	0,483	0,530	0,578	0,629	0,682	21
21,5	0,223	0,255	0,289	0,325	0,363	0,404	0,447	0,492	0,539	0,589	0,640	0,695	21,5
22	0,227	0,260	0,294	0,331	0,370	0,411	0,455	0,501	0,549	0,599	0,652	0,707	22
22,5	0,231	0,264	0,299	0,337	0,376	0,418	0,463	0,509	0,558	0,610	0,663	0,719	22,5
23	0,235	0,269	0,304	0,342	0,383	0,426	0,471	0,518	0,568	0,620	0,674	0,732	23
23,5	0,239	0,273	0,309	0,348	0,389	0,433	0,478	0,527	0,577	0,630	0,686	0,744	23,5
24	0,243	0,277	0,314	0,354	0,395	0,440	0,486	0,535	0,586	0,640	0,697	0,756	24
24,5	0,247	0,282	0,319	0,359	0,402	0,447	0,494	0,544	0,596	0,651	0,708	0,768	24,5
25	0,250	0,286	0,324	0,365	0,408	0,453	0,501	0,552	0,605	0,661	0,719	0,780	25
25,5	0,254	0,291	0,329	0,370	0,414	0,460	0,509	0,560	0,614	0,671	0,730	0,791	25,5
26	0,258	0,295	0,334	0,376	0,420	0,467	0,517	0,569	0,623	0,681	0,741	0,803	26
26,5	0,262	0,299	0,339	0,381	0,426	0,474	0,524	0,577	0,633	0,691	0,752	0,815	26,5
27		0,304	0,344	0,387	0,433	0,481	0,532	0,585	0,642	0,701	0,762	0,827	27
27,5		0,308	0,349	0,392	0,439	0,488	0,539	0,593	0,651	0,711	0,773	0,838	27,5
28		0,312	0,354	0,398	0,445	0,495	0,547	0,602	0,660	0,721	0,784	0,850	28
28,5		0,316	0,358	0,403	0,451	0,501	0,554	0,610	0,669	0,730	0,794	0,862	28,5
29			0,363	0,409	0,457	0,508	0,562	0,618	0,678	0,740	0,805	0,873	29
29,5			0,368	0,414	0,463	0,515	0,569	0,626	0,686	0,750	0,815	0,884	29,5
30				0,419	0,469	0,521	0,576	0,635	0,695	0,759	0,826	0,896	30
30,5				0,425	0,475	0,528	0,584	0,643	0,704	0,769	0,837	0,907	30,5
31				0,430	0,481	0,534	0,591	0,651	0,713	0,778	0,847	0,919	31
31,5				0,435	0,486	0,541	0,598	0,659	0,722	0,788	0,857	0,930	31,5
32					0,493	0,548	0,606	0,667	0,731	0,798	0,868	0,942	32
32,5					0,498	0,554	0,613	0,675	0,739	0,807	0,878	0,953	32,5
33						0,561	0,620	0,682	0,748	0,817	0,888	0,964	33
33,5						0,567	0,627	0,690	0,757	0,826	0,899	0,975	33,5
34								0,698	0,765	0,836	0,909	0,986	34
34,5									0,774	0,845	0,919	0,997	34,5
35										0,854	0,929	1,008	35
35,5										0,863	0,939	1,019	35,5
36										0,873	0,950	1,030	36
36,5										0,882	0,960	1,041	36,5

Table 7 (continued).

Tabelle I (Fortsetzung).

Height Höhe Højde m	Diameter b.h. - Durchmesser - Diameter cm												Height Höhe Højde m
	27	28	29	30	31	32	33	34	35	36	37	38	
	Total volume, cubic meter - Baummassen, fm - Totalmasse, m^3												
13,5	0,526	0,567	0,610	0,654									
14	0,541	0,583	0,627	0,672									
14,5	0,556	0,599	0,644	0,691	0,739	0,789	0,841	0,894	0,950	1,007	1,065	1,127	14,5
15	0,570	0,615	0,661	0,709	0,759	0,809	0,863	0,918	0,975	1,033	1,093	1,155	15
15,5	0,585	0,630	0,678	0,727	0,778	0,831	0,885	0,941	0,999	1,059	1,121	1,184	15,5
16	0,599	0,646	0,695	0,745	0,797	0,851	0,907	0,964	1,024	1,085	1,148	1,214	16
16,5	0,614	0,662	0,711	0,762	0,816	0,871	0,928	0,988	1,048	1,111	1,176	1,243	16,5
17	0,628	0,676	0,727	0,780	0,835	0,891	0,950	1,010	1,072	1,136	1,203	1,271	17
17,5	0,642	0,692	0,744	0,797	0,853	0,911	0,971	1,033	1,097	1,162	1,230	1,299	17,5
18	0,656	0,707	0,760	0,815	0,872	0,931	0,992	1,055	1,121	1,187	1,256	1,328	18
18,5	0,670	0,722	0,776	0,832	0,890	0,951	1,013	1,078	1,144	1,212	1,283	1,355	18,5
19	0,683	0,737	0,792	0,849	0,909	0,970	1,034	1,099	1,167	1,237	1,309	1,383	19
19,5	0,697	0,751	0,808	0,866	0,927	0,990	1,054	1,122	1,191	1,262	1,335	1,411	19,5
20	0,711	0,766	0,823	0,883	0,945	1,009	1,075	1,143	1,214	1,287	1,361	1,439	20
20,5	0,724	0,781	0,839	0,900	0,963	1,028	1,095	1,165	1,237	1,311	1,387	1,466	20,5
21	0,738	0,795	0,855	0,916	0,981	1,047	1,115	1,187	1,260	1,335	1,413	1,493	21
21,5	0,751	0,810	0,870	0,933	0,999	1,066	1,136	1,208	1,283	1,359	1,439	1,520	21,5
22	0,764	0,824	0,886	0,950	1,016	1,085	1,156	1,230	1,306	1,384	1,464	1,547	22
22,5	0,778	0,838	0,901	0,966	1,034	1,104	1,176	1,251	1,328	1,408	1,489	1,574	22,5
23	0,791	0,852	0,916	0,982	1,052	1,123	1,196	1,272	1,351	1,431	1,515	1,601	23
23,5	0,804	0,866	0,931	0,999	1,069	1,141	1,216	1,293	1,373	1,455	1,540	1,627	23,5
24	0,817	0,880	0,946	1,015	1,086	1,160	1,236	1,314	1,396	1,479	1,565	1,653	24
24,5	0,830	0,894	0,962	1,031	1,104	1,179	1,255	1,335	1,418	1,503	1,590	1,680	24,5
25	0,843	0,908	0,976	1,047	1,121	1,196	1,275	1,356	1,439	1,525	1,615	1,706	25
25,5	0,856	0,922	0,991	1,063	1,138	1,215	1,294	1,377	1,461	1,549	1,639	1,732	25,5
26	0,868	0,936	1,007	1,079	1,155	1,233	1,314	1,397	1,484	1,572	1,664	1,758	26
26,5	0,881	0,950	1,021	1,094	1,172	1,251	1,333	1,418	1,505	1,595	1,688	1,784	26,5
27	0,894	0,963	1,035	1,110	1,189	1,269	1,352	1,438	1,527	1,618	1,712	1,809	27
27,5	0,906	0,977	1,051	1,126	1,205	1,287	1,371	1,459	1,548	1,641	1,737	1,834	27,5
28	0,919	0,991	1,065	1,141	1,222	1,305	1,391	1,479	1,570	1,664	1,761	1,860	28
28,5	0,932	1,004	1,079	1,157	1,239	1,323	1,409	1,499	1,592	1,687	1,785	1,886	28,5
29	0,944	1,017	1,094	1,172	1,255	1,340	1,428	1,519	1,613	1,709	1,809	1,911	29
29,5	0,956	1,030	1,108	1,188	1,271	1,358	1,447	1,538	1,633	1,731	1,832	1,936	29,5
30	0,968	1,044	1,122	1,204	1,288	1,375	1,465	1,559	1,655	1,754	1,856	1,961	30
30,5	0,981	1,057	1,136	1,218	1,304	1,393	1,484	1,579	1,675	1,776	1,879	1,986	30,5
31	0,993	1,070	1,151	1,234	1,321	1,410	1,503	1,598	1,697	1,798	1,903	2,010	31
31,5	1,006	1,084	1,165	1,249	1,337	1,428	1,521	1,618	1,718	1,820	1,926	2,035	31,5
32	1,017	1,097	1,179	1,264	1,353	1,445	1,539	1,638	1,739	1,843	1,950	2,060	32
32,5	1,030	1,110	1,193	1,280	1,370	1,462	1,558	1,657	1,759	1,864	1,972	2,085	32,5
33	1,042	1,123	1,207	1,294	1,385	1,479	1,576	1,676	1,780	1,886	1,996	2,108	33
33,5	1,054	1,136	1,221	1,309	1,401	1,496	1,594	1,696	1,801	1,907	2,019	2,133	33,5
34	1,066	1,149	1,235	1,325	1,417	1,514	1,613	1,715	1,821	1,929	2,042	2,158	34
34,5	1,078	1,162	1,249	1,339	1,433	1,531	1,630	1,734	1,841	1,951	2,065	2,182	34,5
35	1,089	1,174	1,262	1,354	1,449	1,547	1,648	1,754	1,862	1,972	2,088	2,206	35
35,5	1,101	1,187	1,276	1,369	1,464	1,564	1,666	1,772	1,881	1,994	2,110	2,230	35,5
36	1,113	1,200	1,290	1,383	1,480	1,581	1,684	1,791	1,902	2,016	2,133	2,253	36
36,5	1,125	1,213	1,304	1,398	1,496	1,598	1,702	1,811	1,922	2,037	2,156	2,277	36,5
37					1,512	1,615	1,720	1,829	1,943	2,058	2,178	2,301	37
37,5					1,528	1,631	1,738	1,848	1,963	2,080	2,201	2,326	37,5
38						1,647	1,755	1,866	1,982	2,101	2,222	2,348	38
38,5						1,664	1,773	1,886	2,002	2,122	2,245	2,372	38,5

Table 7 (continued).

Tabelle I (Fortsetzung).

Height Höhe Højde m	Diameter b.h. - Durchmesser - Diameter cm												Height Höhe Højde m
	39	40	41	42	43	44	45	46	47	48	49	50	
	Total volume, cubic meter - Baummassen, fm - Totalmasse, m^3												
14,5	1,187	1,251											
15	1,218	1,284	1,351	1,420									
15,5	1,249	1,316	1,385	1,456	1,529	1,602	1,678						
16	1,280	1,349	1,419	1,492	1,566	1,642	1,720						
16,5	1,311	1,381	1,453	1,528	1,603	1,681	1,761	1,842	1,926	2,011	2,100		16,5
17	1,340	1,413	1,486	1,563	1,640	1,719	1,801	1,884	1,970	2,057	2,147	2,239	17
17,5	1,371	1,444	1,520	1,598	1,677	1,758	1,842	1,927	2,015	2,103	2,196	2,289	17,5
18	1,400	1,476	1,552	1,632	1,713	1,796	1,881	1,968	2,058	2,149	2,243	2,339	18
18,5	1,430	1,507	1,586	1,667	1,750	1,834	1,921	2,010	2,102	2,195	2,291	2,389	18,5
19	1,459	1,538	1,618	1,701	1,785	1,872	1,961	2,051	2,145	2,240	2,337	2,437	19
19,5	1,489	1,569	1,650	1,735	1,821	1,909	2,000	2,092	2,188	2,285	2,384	2,486	19,5
20	1,518	1,599	1,683	1,769	1,857	1,946	2,039	2,133	2,231	2,329	2,432	2,535	20
20,5	1,546	1,629	1,715	1,803	1,892	1,984	2,078	2,174	2,273	2,373	2,477	2,583	20,5
21	1,575	1,660	1,747	1,836	1,927	2,020	2,116	2,214	2,315	2,417	2,524	2,631	21
21,5	1,604	1,690	1,779	1,869	1,962	2,057	2,155	2,255	2,357	2,461	2,569	2,679	21,5
22	1,632	1,720	1,810	1,903	1,997	2,094	2,193	2,295	2,400	2,505	2,615	2,726	22
22,5	1,661	1,750	1,841	1,936	2,032	2,130	2,231	2,334	2,441	2,548	2,660	2,773	22,5
23	1,689	1,780	1,873	1,968	2,066	2,166	2,269	2,373	2,482	2,592	2,705	2,820	23
23,5	1,717	1,809	1,903	2,001	2,100	2,201	2,306	2,413	2,523	2,635	2,750	2,867	23,5
24	1,745	1,839	1,934	2,033	2,134	2,237	2,344	2,452	2,564	2,678	2,794	2,913	24
24,5	1,772	1,867	1,965	2,066	2,168	2,273	2,381	2,491	2,605	2,720	2,839	2,960	24,5
25	1,800	1,896	1,995	2,098	2,201	2,308	2,417	2,530	2,645	2,761	2,882	3,005	25
25,5	1,827	1,925	2,026	2,130	2,235	2,343	2,455	2,568	2,685	2,804	2,926	3,051	25,5
26	1,855	1,954	2,056	2,162	2,269	2,379	2,492	2,607	2,726	2,846	2,971	3,097	26
26,5	1,882	1,983	2,087	2,193	2,302	2,413	2,528	2,645	2,765	2,888	3,015	3,142	26,5
27	1,909	2,011	2,116	2,225	2,335	2,448	2,565	2,683	2,805	2,929	3,058	3,188	27
27,5	1,936	2,040	2,146	2,257	2,368	2,482	2,601	2,721	2,845	2,971	3,101	3,233	27,5
28	1,963	2,069	2,176	2,288	2,401	2,517	2,637	2,759	2,884	3,012	3,144	3,278	28
28,5	1,990	2,097	2,206	2,319	2,434	2,551	2,673	2,796	2,924	3,054	3,187	3,323	28,5
29	2,016	2,124	2,235	2,350	2,466	2,585	2,708	2,833	2,963	3,094	3,229	3,367	29
29,5	2,042	2,152	2,264	2,380	2,498	2,619	2,744	2,870	3,001	3,133	3,271	3,410	29,5
30	2,069	2,180	2,294	2,411	2,531	2,653	2,779	2,907	3,040	3,174	3,313	3,455	30
30,5	2,095	2,207	2,323	2,442	2,563	2,687	2,815	2,945	3,079	3,215	3,356	3,499	30,5
31	2,121	2,235	2,352	2,473	2,595	2,720	2,849	2,981	3,117	3,256	3,398	3,542	31
31,5	2,147	2,263	2,381	2,502	2,627	2,753	2,884	3,018	3,156	3,295	3,440	3,586	31,5
32	2,174	2,290	2,410	2,534	2,659	2,787	2,920	3,055	3,194	3,336	3,482	3,630	32
32,5	2,199	2,317	2,439	2,563	2,690	2,820	2,955	3,091	3,232	3,375	3,523	3,673	32,5
33	2,225	2,344	2,467	2,593	2,722	2,853	2,989	3,127	3,270	3,414	3,564	3,715	33
33,5	2,250	2,371	2,496	2,623	2,753	2,886	3,023	3,163	3,308	3,454	3,605	3,758	33,5
34	2,276	2,399	2,524	2,653	2,785	2,920	3,058	3,200	3,346	3,494	3,647	3,802	34
34,5	2,302	2,426	2,552	2,683	2,816	2,952	3,092	3,235	3,383	3,532	3,687	3,844	34,5
35	2,328	2,452	2,580	2,713	2,847	3,984	3,127	3,271	3,420	3,572	3,728	3,887	35
35,5	2,353	2,478	2,608	2,742	2,877	3,017	3,160	3,306	3,457	3,610	3,768	3,928	35,5
36	2,378	2,505	2,636	2,772	2,908	3,049	3,194	3,342	3,495	3,649	3,809	3,971	36
36,5	2,404	2,533	2,664	2,801	2,940	3,082	3,228	3,378	3,532	3,688	3,849	4,013	36,5
37	2,429	2,559	2,692	2,830	2,971	3,113	3,262	3,412	3,568	3,727	3,890	4,055	37
37,5	2,454	2,585	2,720	2,860	3,001	3,146	3,296	3,448	3,606	3,765	3,930	4,097	37,5
38	2,478	2,611	2,747	2,888	3,031	3,178	3,329	3,483	3,642	3,803	3,969	4,138	38
38,5	2,503	2,638	2,775	2,918	3,062	3,210	3,362	3,518	3,679	3,841	4,009	4,180	38,5
39	2,528	2,663	2,803	2,947	3,092	3,242	3,397	3,553	3,715	3,880	4,049	4,222	39
39,5	2,553	2,690	2,830	2,975	3,123	3,273	3,429	3,588	3,752	3,918	4,089	4,263	39,5

Table 7 (continued).

Tabelle I (Fortsetzung).

Height Höhe Højde m	Diameter b.h. - Durchmesser - Diameter cm												Height Höhe Højde m
	51	52	53	54	55	56	57	58	59	60	61	62	
	Total volume, cubic meter - Baummassen, fm - Totalmasse, m^3												
17	2,331	2,428	2,525	2,624	2,725	2,829	2,934	3,041					
17,5	2,384	2,482	2,582	2,684	2,786	2,893	3,001	3,109					
18	2,435	2,535	2,638	2,742	2,846	2,955	3,065	3,177					
18,5	2,487	2,589	2,694	2,800	2,906	3,018	3,131	3,244					
19	2,538	2,642	2,749	2,857	2,967	3,079	3,194	3,311					
19,5	2,589	2,695	2,804	2,914	3,026	3,141	3,258	3,377					
20	2,639	2,748	2,859	2,971	3,084	3,203	3,322	3,443					
20,5	2,690	2,800	2,913	3,028	3,143	3,263	3,386	3,508					
21	2,740	2,852	2,968	3,084	3,202	3,324	3,448	3,573					
21,5	2,789	2,904	3,021	3,140	3,260	3,385	3,511	3,639					
22	2,839	2,956	3,075	3,196	3,318	3,445	3,573	3,703	3,835	3,972	4,111	4,250	22
22,5	2,888	3,007	3,128	3,251	3,375	3,504	3,635	3,767	3,901	4,041	4,182	4,323	22,5
23	2,937	3,058	3,182	3,306	3,432	3,564	3,697	3,830	3,967	4,110	4,253	4,396	23
23,5	2,985	3,108	3,233	3,360	3,489	3,622	3,758	3,894	4,033	4,178	4,323	4,469	23,5
24	3,034	3,159	3,286	3,415	3,545	3,681	3,819	3,957	4,098	4,245	4,393	4,542	24
24,5	3,082	3,209	3,339	3,470	3,602	3,740	3,880	4,021	4,164	4,313	4,464	4,614	24,5
25	3,130	3,258	3,390	3,523	3,658	3,797	3,939	4,083	4,228	4,379	4,532	4,685	25
25,5	3,178	3,309	3,442	3,576	3,714	3,856	3,999	4,145	4,295	4,446	4,601	4,757	25,5
26	3,226	3,358	3,494	3,631	3,770	3,914	4,060	4,207	4,357	4,513	4,671	4,828	26
26,5	3,273	3,407	3,544	3,684	3,825	3,971	4,119	4,269	4,421	4,579	4,739	4,899	26,5
27	3,320	3,456	3,596	3,736	3,880	4,028	4,178	4,330	4,484	4,645	4,807	4,969	27
27,5	3,367	3,505	3,647	3,789	3,935	4,085	4,237	4,391	4,548	4,711	4,875	5,039	27,5
28	3,413	3,554	3,698	3,842	3,990	4,142	4,296	4,452	4,611	4,777	4,943	5,109	28
28,5	3,460	3,602	3,748	3,895	4,044	4,198	4,355	4,513	4,674	4,841	5,011	5,180	28,5
29	3,506	3,650	3,797	3,946	4,097	4,254	4,413	4,573	4,736	4,906	5,077	5,248	29
29,5	3,552	3,698	3,847	3,998	4,150	4,309	4,470	4,632	4,797	4,969	5,143	5,316	29,5
30	3,597	3,745	3,897	4,049	4,204	4,365	4,528	4,693	4,860	5,034	5,210	5,385	30
30,5	3,643	3,793	3,946	4,101	4,258	4,421	4,586	4,752	4,922	5,098	5,276	5,454	30,5
31	3,689	3,840	3,996	4,152	4,311	4,476	4,643	4,812	4,983	5,162	5,342	5,522	31
31,5	3,734	3,888	4,045	4,205	4,364	4,531	4,700	4,871	5,044	5,225	5,408	5,590	31,5
32	3,780	3,936	4,094	4,255	4,418	4,587	4,758	4,931	5,106	5,289	5,474	5,659	32
32,5	3,825	3,982	4,143	4,305	4,470	4,641	4,814	4,989	5,167	5,352	5,539	5,725	32,5
33	3,869	4,028	4,191	4,355	4,522	4,695	4,870	5,047	5,227	5,414	5,603	5,792	33
33,5	3,914	4,075	4,239	4,406	4,574	4,749	4,926	5,105	5,286	5,476	5,668	5,859	33,5
34	3,959	4,122	4,288	4,457	4,627	4,804	4,983	5,164	5,348	5,540	5,734	5,927	34
34,5	4,003	4,168	4,336	4,506	4,679	4,858	5,038	5,222	5,408	5,602	5,797	5,992	34,5
35	4,047	4,214	4,384	4,556	4,730	4,911	5,095	5,279	5,468	5,664	5,861	6,059	35
35,5	4,091	4,259	4,431	4,605	4,781	4,964	5,149	5,335	5,526	5,724	5,924	6,123	35,5
36	4,135	4,305	4,479	4,654	4,833	5,018	5,205	5,394	5,586	5,787	5,988	6,190	36
36,5	4,179	4,351	4,527	4,704	4,884	5,071	5,260	5,451	5,646	5,848	6,052	6,256	36,5
37	4,223	4,396	4,574	4,753	4,935	5,124	5,315	5,508	5,704	5,909	6,115	6,322	37
37,5	4,267	4,442	4,621	4,803	4,987	5,177	5,370	5,565	5,764	5,970	6,179	6,387	37,5
38	4,309	4,486	4,668	4,850	5,036	5,229	5,424	5,621	5,821	6,030	6,240	6,451	38
38,5	4,353	4,532	4,715	4,900	5,088	5,282	5,479	5,678	5,881	6,091	6,304	6,516	38,5
39	4,396	4,577	4,762	4,948	5,138	5,334	5,534	5,735	5,939	6,151	6,366	6,581	39
39,5	4,439	4,621	4,808	4,997	5,188	5,387	5,587	5,791	5,996	6,212	6,428	6,645	39,5

Table 7 (continued).

Tabelle I (Fortsetzung).

Height Hohe Højde m	Diameter b.h. - Durchmesser - Diameter cm												Height Hohe Højde m
	63	64	65	66	67	68	69	70	71	72	73	74	
	Total volume, cubic meter - Baummassen, fm - Totalmasse, m^3												
22	4,392	4,538	4,685	4,834	4,989								
22,5	4,468	4,616	4,766	4,917	5,074								
23	4,544	4,695	4,846	5,001	5,161	5,320	5,484	5,648	5,817	5,985	6,159	6,334	23
23,5	4,618	4,772	4,926	5,083	5,246	5,408	5,574	5,741	5,913	6,084	6,260	6,439	23,5
24	4,694	4,849	5,006	5,165	5,331	5,496	5,665	5,834	6,009	6,183	6,362	6,543	24
24,5	4,769	4,928	5,086	5,248	5,417	5,583	5,756	5,928	6,105	6,282	6,464	6,648	24,5
25	4,841	5,003	5,164	5,328	5,500	5,669	5,844	6,019	6,198	6,378	6,563	6,750	25
25,5	4,916	5,079	5,243	5,410	5,583	5,756	5,933	6,111	6,294	6,476	6,664	6,853	25,5
26	4,990	5,156	5,322	5,492	5,668	5,842	6,023	6,203	6,389	6,573	6,764	6,957	26
26,5	5,062	5,231	5,400	5,572	5,751	5,928	6,111	6,294	6,482	6,670	6,863	7,058	26,5
27	5,135	5,307	5,477	5,652	5,833	6,013	6,198	6,384	6,575	6,765	6,961	7,160	27
27,5	5,209	5,381	5,556	5,732	5,916	6,098	6,287	6,474	6,668	6,861	7,060	7,261	27,5
28	5,281	5,456	5,633	5,811	5,998	6,183	6,374	6,564	6,761	6,957	7,158	7,362	28
28,5	5,353	5,531	5,709	5,891	6,080	6,267	6,461	6,655	6,853	7,052	7,256	7,463	28,5
29	5,424	5,604	5,785	5,969	6,160	6,351	6,546	6,742	6,944	7,145	7,352	7,561	29
29,5	5,494	5,677	5,860	6,047	6,240	6,433	6,631	6,830	7,034	7,238	7,447	7,660	29,5
30	5,565	5,751	5,936	6,125	6,322	6,516	6,717	6,918	7,125	7,332	7,544	7,759	30
30,5	5,636	5,824	6,012	6,203	6,401	6,600	6,803	7,006	7,216	7,425	7,640	7,858	30,5
31	5,707	5,896	6,087	6,281	6,482	6,682	6,888	7,095	7,306	7,518	7,736	7,956	31
31,5	5,777	5,969	6,161	6,357	6,561	6,764	6,973	7,181	7,396	7,610	7,831	8,054	31,5
32	5,848	6,042	6,237	6,436	6,642	6,847	7,059	7,270	7,486	7,703	7,927	8,153	32
32,5	5,917	6,114	6,311	6,512	6,720	6,928	7,142	7,355	7,575	7,795	8,020	8,249	32,5
33	5,985	6,184	6,384	6,587	6,798	7,008	7,224	7,441	7,663	7,885	8,113	8,345	33
33,5	6,055	6,256	6,458	6,664	6,877	7,090	7,307	7,527	7,752	7,976	8,207	8,441	33,5
34	6,125	6,328	6,533	6,741	6,957	7,172	7,392	7,614	7,842	8,069	8,302	8,539	34
34,5	6,193	6,398	6,606	6,816	7,034	7,251	7,475	7,698	7,929	8,158	8,395	8,634	34,5
35	6,262	6,470	6,679	6,891	7,112	7,332	7,558	7,784	8,017	8,249	8,488	8,730	35
35,5	6,328	6,539	6,750	6,965	7,188	7,409	7,638	7,867	8,102	8,336	8,578	8,823	35,5
36	6,397	6,610	6,824	7,040	7,266	7,490	7,721	7,952	8,190	8,428	8,672	8,919	36
36,5	6,465	6,680	6,896	7,115	7,343	7,570	7,804	8,037	8,278	8,517	8,764	9,014	36,5
37	6,533	6,750	6,968	7,189	7,420	7,649	7,885	8,121	8,364	8,606	8,855	9,108	37
37,5	6,601	6,820	7,040	7,265	7,497	7,729	7,967	8,205	8,451	8,696	8,947	9,202	37,5
38	6,667	6,888	7,111	7,337	7,572	7,806	8,046	8,287	8,535	8,782	9,037	9,294	38
38,5	6,735	6,958	7,183	7,411	7,649	7,885	8,129	8,372	8,622	8,872	9,129	9,389	38,5
39	6,801	7,027	7,254	7,485	7,725	7,963	8,209	8,455	8,708	8,960	9,219	9,482	39
39,5	6,867	7,096	7,324	7,558	7,800	8,041	8,289	8,537	8,792	9,047	9,309	9,574	39,5

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